

Compton effect - energy-dispersive direct measurement



Physics

Modern Physics

Production & use of X-rays



Difficulty level

hard



Group size

2



Preparation time

45+ minutes



Execution time

45+ minutes

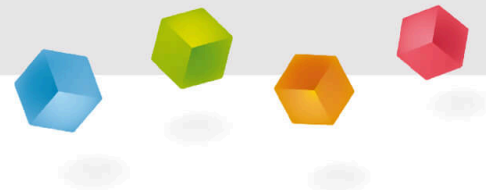
This content can also be found online at:



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General information



Application

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Setup

Most applications of X rays are based on their ability to pass through matter. Since this ability is dependent on the density of the matter, imaging of the interior of objects and even people becomes possible. This has wide usage in fields such as medicine or security.

Other information (1/2)

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**Prior****knowledge****Main****principle**

The prior knowledge for this experiment is found in the Theory section.

Photons of the molybdenum K_{α} X-ray line are scattered at the quasi-free electrons of an acrylic glass cuboid. The energy of the scattered photons is determined in an angle-dependent manner with the aid of a swivelling semiconductor detector and a multi-channel analyser.

Other information (2/2)

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**Learning
objective****Tasks**

The goal of this experiment is to get to investigate the Compton effect.

1. Calibrate the semiconductor energy detector.
2. Energy determination of the photons of the $W-L_{\alpha}$ -line that are scattered through an acrylic glass element as a function of the scattering angle.
3. Compare the measured energy values of the lines of scatter with the calculated energy values.
4. Calculate the Compton wavelength of electrons and a comparison of this value with the corresponding value of the 90° scattering.

Theory (1/2)

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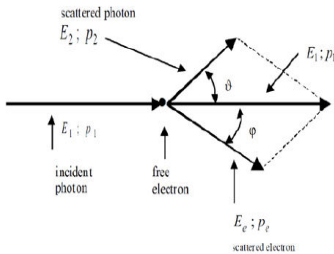


Fig. 1: Momentum and energy relation of the Compton effect

Figure 1 shows a schematic representation of the Compton Effect.

Due to the interaction with a free electron in the solid element, the incident photon loses energy and is deflected from its original direction with the scattering angle θ . The previously resting electron absorbs additional kinetic energy and leaves the collision point under the angle ϕ .

$$E_2 = \frac{E_1}{1 + \frac{E_1}{m_e c^2} (1 - \cos(\theta))} \quad (1)$$

Photon energy before and after the collision E_1 resp. E_2

Equivalent $1 \text{ eV} = 1.6021 \cdot 10^{-19} \text{ J}$

Scattering angle θ

Speed of light in vacuum

$$c = 2.988 \cdot 10^8 \text{ m/s}$$

Rest mass of the electron

$$m_e = 9.109 \cdot 10^{-31} \text{ kg}$$

Theory (2/2)

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After the collision, the photon has a lower energy level E_2 and a higher wavelength λ_2 . With $E = h \cdot \nu$, (1) can be transformed into:

$$\frac{1}{h\nu_2} - \frac{1}{h\nu_1} = \frac{1}{m_e c^2} (1 - \cos(\theta)) \quad (2)$$

With $\lambda = c/\nu$, (2) leads to:

$$\lambda_2 - \lambda_1 = \frac{h}{m_e c} (1 - \cos(\theta)) \quad (3)$$

For the 90° scattering, the difference in wavelength, which only consists of the three universal components, provides the so-called Compton wavelength λ_C for electrons.

$$\lambda_C = \frac{h}{m_e c} = 2.426 \text{ pm}$$

As far as the special cases of the forward and backward scattering by $\theta = 0^\circ$ and $\theta = 180^\circ$ are concerned, the change in wavelength is $\Delta\lambda = 2\lambda_C$.

Planck's quantum of action $h = 6.626 \cdot 10^{-34} \text{ Js}$

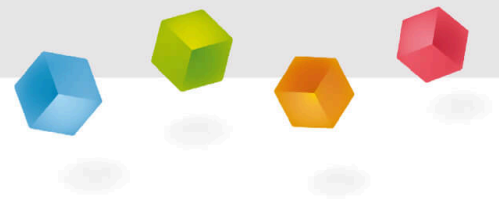
Photon frequency ν

Equipment

Position	Material	Item No.	Quantity
1	XR 4.0 expert unit, 35 kV	09057-99	1
2	XR 4.0 X-ray goniometer	09057-10	1
3	XR4 X-ray Plug-in Cu tube	09057-51	1
4	XR 4.0 X-ray material upgrade set	09165-88	1

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Setup and Procedure



Setup (1/2)

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- Screw the adapter ring onto the inlet tube of the energy detector and connect the signal and supply cables to the corresponding ports of the detector with the aid of the right-angle plugs.
- Connect the signal and supply cables to the corresponding ports in the experiment chamber of the X-ray unit. In Figure 2, the port for the signal cable is marked in red and the port for the supply cable is marked in green. Connect the external X RED ports of the x-ray unit (see Fig. 3) to the multi-channel analyser (MCA). Connect the signal cable to the "Input" port and the supply cable to the "X-Ray Energy Det." port of the MCA.

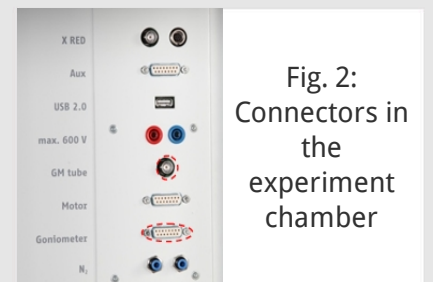


Fig. 2:
Connectors in
the
experiment
chamber



Fig. 3: Connection of the
multi-channel analyser

Setup (2/2)

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- Secure the energy detector in the holder of the swivel arm of the goniometer. Lay the two cables with sufficient length so that the goniometer can be swivelled freely over the entire range.
- Connect the multi-channel analyser and computer with the aid of the USB cable.
- Insert the tube with the 2-mm-aperture.
- Bring the goniometer block and the detector to their respective end positions on the right.

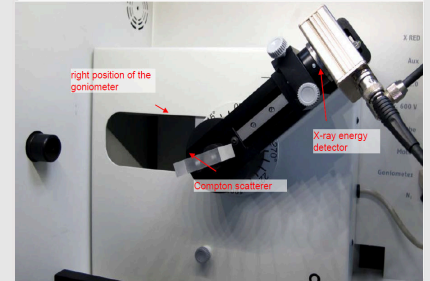


Fig. 4: Goniometer set-up

Procedure (1/5)

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- Bring the goniometer block and the detector to their respective end positions on the right.
- Insert the tube with the 1mm-aperture into the exit tube of the X-ray tube.
- With the X-ray unit switched on and the door locked, bring the detector to the 0° position. Then, shift the detector by some tenths degree out of the zero position in order to reduce the total rate.
- Operating data of the tungsten X-ray tube: Select an anode voltage $U_A = 25$ kV and an anode current $I_A = 0.02$ mA and confirm these values by pressing the "Enter" button.
- Switch on the X-radiation

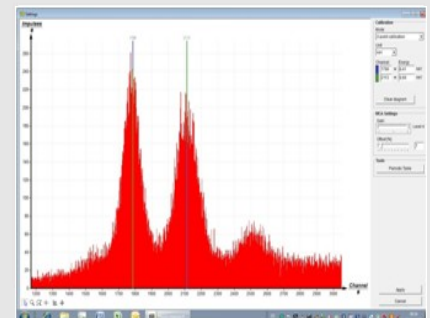


Fig. 5: calibration of the multi-channel analyser

Procedure (2/5)

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- In the MEASURE program, select "Multi channel analyser" under "Gauge". Then, select "Settings and calibration". After the "Calibrate" button has been clicked, a spectrum can be measured. The counting rate should be < 300 c/s. Energy calibration settings: - 2-point calibration, - Unit = keV, Gain = 2 – Set the offset so that low-energy noise signals will be suppressed (usually a few per cent are sufficient), See Fig 5.
- Measuring time: 5 minutes. Use the timer of the X-ray unit.
- Make the two coloured calibration lines congruent with the line centres of the two characteristic X-ray lines. The corresponding energy values (see e.g. P2544705) $E(L_3M_5/L_3M_4) = 8,41\text{keV}$ and $E(L_2N_4) = 9,69\text{keV}$ are entered into the corresponding fields, depending on the colour. (Note: Since a separation of the lines L_3M_4 and L_3M_5 Lines is not possible, the mean value of both lines is entered as the energy of the line).
- Name and save the calibration.

Procedure (3/5)

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Compton scattering

Set the detector to the zero position and select the following operating data: diaphragm tube with $d = 1\text{mm}$, $U_A = 30\text{ kV}$, $I_A = 0.08\text{ mA}$.

- Enter the following parameters into the field "Control" in the window "Spectra recording": - Gain = 2, - Offset = 5%, - X-Data = keV, - Interval width [channels] = 1.
- Start the X-ray tube. The measuring time should be approximately 5 minute so that the intensity of the K_α -peak is approximately 200-300 pulses. Accept the data and save them.
- Place the acrylic glass element (scatterer) of the Compton equipment into the sample holder and set it to a 10° position. Set the detector to 20° .

Procedure (4/5)






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- Now, add the tube with the 5 mm aperture and increase the operating data of the X-ray tube to $U_A = 35$ kV and $I_A = 0.3$ mA.
- Start the measurement. The measuring time is approximately 10 minutes. The intensity of the K_α -peak should be approximately 200 pulses. Stop the measurement with "Accept data".
- Leave the acrylic glass scatterer in its position and perform additional measurements. To do so, change the angle of the detector in steps of 10° up to the final value of 160° .

Procedure (5/5)

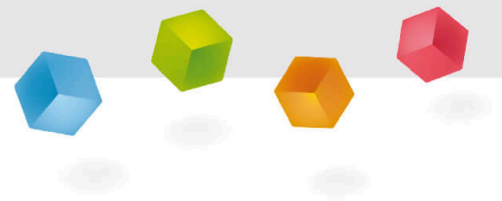
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Evaluation of the measurement curves

- In order to determine the line energy, switch from the bar display to the curve display. To do so, click "Display options" and then "Interpolation and straight lines".
- Extend the relevant line section with the aid of the zoom function 
- Then, select the curve section with  Open the window "Function fitting"  Then, select "Scaled normal distribution" and confirm.
- Find the line centroid of the normal distribution with "Peak analysis"  or determine it with the function "Survey" 

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Evaluation



Task 2

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Figure 6 shows a part of the X-ray spectrum of molybdenum. For the angle-dependent displacement of energy of the scattered radiation, only the high-intensity L_{α} -line is to be taken into consideration.

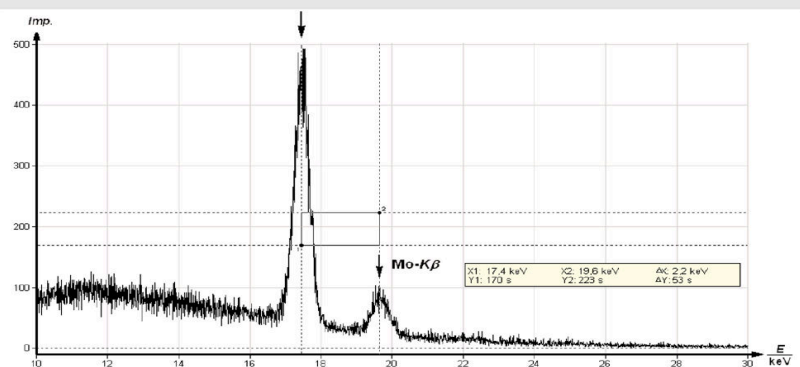


Fig. 6: X-ray spectrum of molybdenum (section)

Task 3

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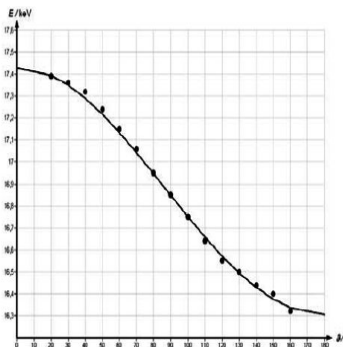


Fig. 7: Energy of the molybdenum K_{α} -line as a function of the scattering angle.

Column B of table 1 shows the experimental energy values of the line peaks of the $W-L_{\alpha}$ -line as a function of the scattering angle (column A).

For comparison, the column C shows the energy values that were calculated with $E_1(\text{Mo} - K_{\alpha}) = 17.43 \text{ keV}$ based on (1).

Figure 7 shows the content of table 1 in graphical form for clarification.

A	B	C
$\theta / ^{\circ}$	$E_2 (\text{exp.}) / \text{keV}$	$E_2 (\text{theor.}) / \text{keV}$
20	17,39	17,394
30	17,36	17,350
40	17,32	17,290
50	17,24	17,218
60	17,15	17,134
70	17,06	17,043
80	16,95	16,947
90	16,85	16,849
100	16,75	16,752
110	16,64	16,659
120	16,55	16,572
130	16,50	16,495
140	16,44	16,429
150	16,40	16,376
160	16,32	16,337

Table 1: Energy E_2 of the scattered photons as a function of the scattering angle θ .

Task 4

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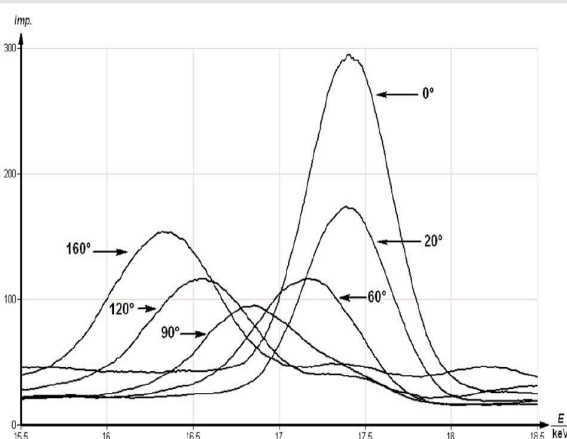


Fig. 8: Molybdenum- K_{α} -Line of various scattering angles θ .

In order to determine the Compton wavelength λ_C based on the 90° scattering, equation (3) is transformed with $\lambda = hc/E$:

$$\lambda_C = \lambda_2 - \lambda_1 = hc \left(\frac{1}{E_2} - \frac{1}{E_1} \right) \quad (4)$$

With $E_2(90^{\circ}) = 16.64 \text{ keV}$ (see table) and $E_1(0^{\circ}) = 17.43 \text{ keV}$ and the equivalence $1 \text{ eV} = 1.602 \cdot 10^{-19} \text{ J}$, one obtains the following Compton wavelength based on the experiment:

$$\lambda_C = 2.49 \text{ pm}$$

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Appendix

Conservation of momentum:

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$$p_1 = p_2 + p_3 \rightarrow p_e^2 = p_1^2 + p_2^2 - 2p_1p_2 \quad (5)$$

The following applies to the angle θ that is formed by the two momentum vectors p_1 and p_2 :

$$\cos(\theta) = \frac{p_1 p_2}{\sqrt{|p_1|^2 |p_2|^2}} \quad (6)$$

With (6) and the momentum-energy relations $p_1 = E_1/c$ and $p_2 = E_2/c$ (momentum-energy relation based on a combination of $E = h\nu$, the photon momentum $p = h/\lambda$ (de Broglie) and $c = \lambda\nu$), equation (5) leads to:

$$p_e^2 = \frac{1}{c^2} (E_1^2 + E_2^2 - 2E_1E_2 \cos(\theta)) \quad (7)$$

Conservation of energy:

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If one takes the relativistic effects for an electron with the velocity v into consideration, the following results:

$$E_1 + m_0c^2 = E_2 + E_e = E_2 + \frac{m_0c^2}{\sqrt{1-v^2/c^2}} \quad (8)$$

With $E = mc^2$ and $p_e = mv$ it follows:

$$v^2 = \frac{c^4 p_e^2}{E_e^2} \quad (9)$$

If one puts (9) into (8), the following results:

$$p_e^2 = \frac{1}{c^2} (E_1^2 + E_2^2 + 2m_0c^2(E_1 - E_2) - 2E_1E_2) \quad (10)$$

The combination of (7) and (10) leads to the following for E_2 :

$$E_2 = \frac{E_1}{1 + \frac{E_1}{m_0c^2} (1 - \cos(\theta))} \quad (11)$$