## **RLC** measuring bridge





http://localhost:1337/c/6009d4c16080870003ab89b7





# **General information**

## **Application**

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Fig. 1: Experimental set-up for determining unknown inductances

Coils, Resistors and Capacitors are a fundamental parts of electrical circuits. As such knowlege about their properties under electrical currents is importent for the understanding of their effects on such circuits.





## Other information (2/2)

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## **Theory (1/6)**

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In principal an unknown resistance can be determined by measuring current and voltage across the resistance. The finite intrinsic resistances of the instruments would introduce significant errors. To avoid such errors the measurement has to be done current-free.

In a Wheatstone bridge circuit the unknown resistance  $R_x$  is connected two three known resistances (see Fig. 5) of which at least one is variable. In this experiment  $R_1$  and  $R_2$  are adjusted in such manner that no current is flowing through the instrument G (alignment of the bridge) which means the voltage across G vanishes as well. In this case the voltages across  $R_x$ and  $R_1$  are equal as well as across  $R_2$  and  $R_3$ . Also, as no current is flowing through G, the same current is flowing through  $R_x$  and  $R_3$  on the one hand (denoted as  $I_1$ ) and through  $R_1$  and  $R_2$  on the other hand ( $I_2$ ). This results into the following equations:

 $I_1R_x=I_2R_1$  and  $I_1R_3=I_2R_2$ 

## Theory (2/6)

Division of these relations yields eq. (4) which computes the unknown resistance  $R_x$ :

$$R_x = R_3 \cdot rac{R_1}{R_2}$$

The voltage at the source is unimportant and may even be time-dependent. In the general bridge construction we have impedances  $Z_x$  and  $Z_3$  instead of the resistances  $R_x$  and  $R_3$ . The current in the middle branch is zero when

$$rac{R_1}{R_2}=rac{Z_3}{Z_x}$$
 (1)

In complex notation, the general form of the impedance Z is given by eq. (2) where i is the complex number,  $\omega = 2\pi\nu$  is the cyclic frequency and  $\nu$  is the frequency of the applied voltage.

$$Z=R+i\left(\omega L-rac{1}{\omega C}
ight)$$
 (2)





Fig. 2: Schematic circuit of the Wheatstone bridge

## **Theory (3/6)**

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Substituting into (1) gives relation (3):

$$rac{R_1}{R_2} = rac{R_3 + i \left(\omega L_3 - rac{1}{\omega C_3}
ight)}{R_x + i \left(\omega L_x - rac{1}{\omega C_x}
ight)}$$
 (3)

The real and imaginary parts must agree on both sides. Therefore we obtain two conditions. The first is the Amplitude condition eq. (4):

# $\frac{R_1}{R_2} = \frac{R_x R_3 + \left(\omega L_3 - \frac{1}{\omega C_3}\right) \left(\omega L_x - \frac{1}{\omega C_x}\right)}{R_x^2 + \left(\omega L_x - \frac{1}{\omega C_x}\right)} \quad (4)$

## Theory (4/6)

the second is the phase condition eq. (5):

$$R_3\left(\omega L_x-rac{1}{\omega C_x}
ight)=R_x\left(\omega L_3-rac{1}{\omega C_3}
ight)$$
 (5)

To fulfill (5),  $Z_x$  and  $Z_3$  must consist of components that are alike.

In the case of  $Z_x$  and  $Z_3$  being ohmic resistances, (1) simplifies to

$$rac{R_1}{R_2}=rac{R_3}{R_x}$$
 (6)

The ratio  $R_1/R_2$  is given by the length x and the total length l of the slide wire (Fig. 2). We thus obtain the unknown resistance with relation (7)

$$R_x = R_3 \cdot \frac{x}{l-x} \quad (7)$$

where  $R_1 = l - x$  and  $R_2 = x$ 

Besides an inductive ressitance, coils also have an ohmic reactance:

$$egin{aligned} Z_x &= R_x + i\omega L_x \ Z_3 &= R_3 + i\omega L_3 \end{aligned}$$

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## **Theory (5/6)**

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From (5) we first obtain relation (8)  $\frac{R_3}{R_x} = \frac{L_3}{L_x}$  (8)

In order to be able to fulfill this condition, we have to connect an additional resistance into the bridge branch with the coils (see Fig. 4)  $R_x$  and  $R_3$  is then the sum of the additional resistance and the ohmic resistance of the coil in the corresponding branch. Substituting (8) into (5) we obtain

$$\frac{R_1}{R_2} = \frac{L_3}{L_x}$$

so that relation (9) holds:

 $L_x = L_3 rac{x}{l-x}$  (9)

## Theory (6/6)

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The inductance of the reference coil (Art. No. <u>11006-01</u>) is determined from the dimensions:

$$L_3 = 2.1 \cdot 10^{-6} \cdot N^2 \cdot r \cdot (rac{r}{l})^{3/4}$$

where L is in H, radius r and length l are in m, and N is the number of turns. For the given coil one obtains  $L_3 = 0.8$  mH.

For capacitances we obtain from relation (5)

$$\frac{R_1}{R_2} = \frac{C_4}{C_3}$$

and thus

$$C_x = C_3 rac{l-x}{x}$$
 (10)



## Equipment

Position	Material	Item No.	Quantity
1	PHYWE Digital Function Generator, USB	13654-99	1
2	Slide wire measurement bridge	07182-00	1
3	Headphone, stereo	65974-00	1
4	Coil, 300 turns	06513-01	1
5	Coil, 600 turns	06514-01	1
6	Coil, 1200 turns	06515-01	1
7	Coil, 600 turns, short	06522-01	1
8	Induction coil,300 turns,dia.40mm	11007-01	1
9	Resistor 1 Ohm 2%, 2W, G1	06055-10	1
10	Resistor 2 Ohm 5%, 2W, G1	06055-20	1
11	Resistor 5 Ohm 2%, 2W, G1	06055-50	1
12	Resistor 10 Ohm 2%, 2W, G1	06056-10	1
13	Resistor 330 Ohm, 1W, G1	39104-13	1
14	Resistor 470 Ohm, 1W, G1	39104-15	1
15	Resistor 680 Ohm, 1W, G1	39104-17	1
16	Resistor 1 kOhm, 1W, G1	39104-19	2
17	Resistor 1,5 kOhm, 1W, G1	39104-21	1
18	Resistor 2.2 kOhm, 1W, G1	39104-23	1
19	Resistor 3.3 kOhm, 1W, G1	39104-25	1
20	Capacitor 100pF/100V, G1	39105-04	1
21	Capacitor 470pF/100V, G1	39105-07	1
22	Capacitor 1nF/ 100V, G1	39105-10	1
23	Capacitor 47nF/ 250V, G1	39105-17	1
24	Capacitor 10nF/ 250V, G1	39105-14	1
25	Capacitor 100 nF/250V, G1	39105-18	1
26	Connection box	06000-00	1
27	Connecting cord, 32 A, 250 mm, red	07360-01	2
28	Connecting cord, 32 A, 250 mm, blue	07360-04	2
29	Connecting cord, 32 A, 750 mm, red	07362-01	2
30	Connecting cord, 32 A, 750 mm, yellow	07362-02	1
31	Short-circuit plug, white	06027-06	2
32	Connecting cord, 32 A, 750 mm, blue	07362-04	2
33	Headphone Adapter jack plug/2 x 4 mm plug	65974-01	1



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## **Setup and Procedure**

## Setup (1/2)

 $\circ\,$  Set up the experiment according to Fig. 1 and the respective schematic in Fig. 3–5. the digital frequency generator is connected in parallel to the measuring bridge. For the measurement of resistances use the resistance of  $1.0\,k\Omega\,(39104-19)$  as reference. For the measurement of the inductances the variable resistance is needed to compensate for the different DC resistances in the coils. Use the induction coil (11006-01) as reference. The Headphone is used instead of an Ampère-meter to balance the Wheatstone circuit.





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## Procedure

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• Choose a frequency on the frequency generator that is suited for the measurements, e.g. 400 Hz. Tune the amplitude to no more than 1V before putting on the headphones. In case of the measurement of inductances, try and find that resistance  $R_P$  for which the sound at a given slide position is lowest. Move the slide into each direction until the sound minimizes or disappears entirely. Especially for the inductance measurement, only a range can be determined, for which the sound is lowest. Try to determine that range as carefully as possible and note down the medium point of that range as the distance with lowest amplitude. When the sound amplitude becomes too low to hear, tune up the voltage output to a maximum of 20 V. Do not forget to tune it down as soon as you have found the balancing point and before changing any part of the circuit. Do not switch resistances/coils/capacitances while having such a high signal amplitude with headphones on – you are in danger of damaging your eardrums seriously!

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## **Evaluation**

## Task 1

In the following the evaluation of the obtained values is described with the help of example values. Your results may vary from those presented here. For Task 1, reference resistance  $1.0 \text{ k}\Omega$  was used. In Tab. 1 the measured values are shown and the calculated resistances are compared to the expected values. Most values compare quite well, only the values for Art. No. <u>39104-21</u> deviate from the expectation.

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Resistor (Art. N	lo.)x [mm] $R$	$t_x$ [ $\Omega$ ] (calc	.) $R_x$ [ $\Omega$ ] (theo.)
39104-13	244	323	330
39104-15	317	464	470
39104-17	402	672	680
39104-19	502	1008	1000
39104-21	616	1604	1500
39104-23	688	2205	2200
39104-25	768	3310	3300

Tab. 1: Measured and calculated values for Task 1.

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## Task 2

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In Tab. 2 the measured values of Task 2 are shown and the calculated inductances are compared to the expected values. The values compare quite well.

Inductance (Art.	X	$R_P$ [	$L_x$ [mH]	$L_x$ [mH]
No.)	[mm]	Ω]	(calc.)	(theo.)
06513-01	620	1	1.3	2
06514-01	920	2	9.2	9
06515-01	975	10	31.2	35
06522-01	950	5	15.2	15

Tab. 2: Measured and calculated values for Task 2.  $R_P$  denotes the additional resistance that compensates the coil's DC resistance.

## Task 3

For Task 3 the capacitance of 1 nF (<u>39105-10</u>) had been used as reference. The measured values are shown in Tab. 3 as well as calculated and expected capacitances. The experimental values compare very well with theoretical values.

#### Capacitance (Art. $C_x$ [nF] $C_x$ [nF] Х No.) [mm] (calc.) (theo.) 39105-04 910 0.10 0.1 39105-07 680 0.47 0.47 39105-14 100 9.00 10 39105-18 5 99.00 100

Tab. 3: Measured and calculated values for Task 3.

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