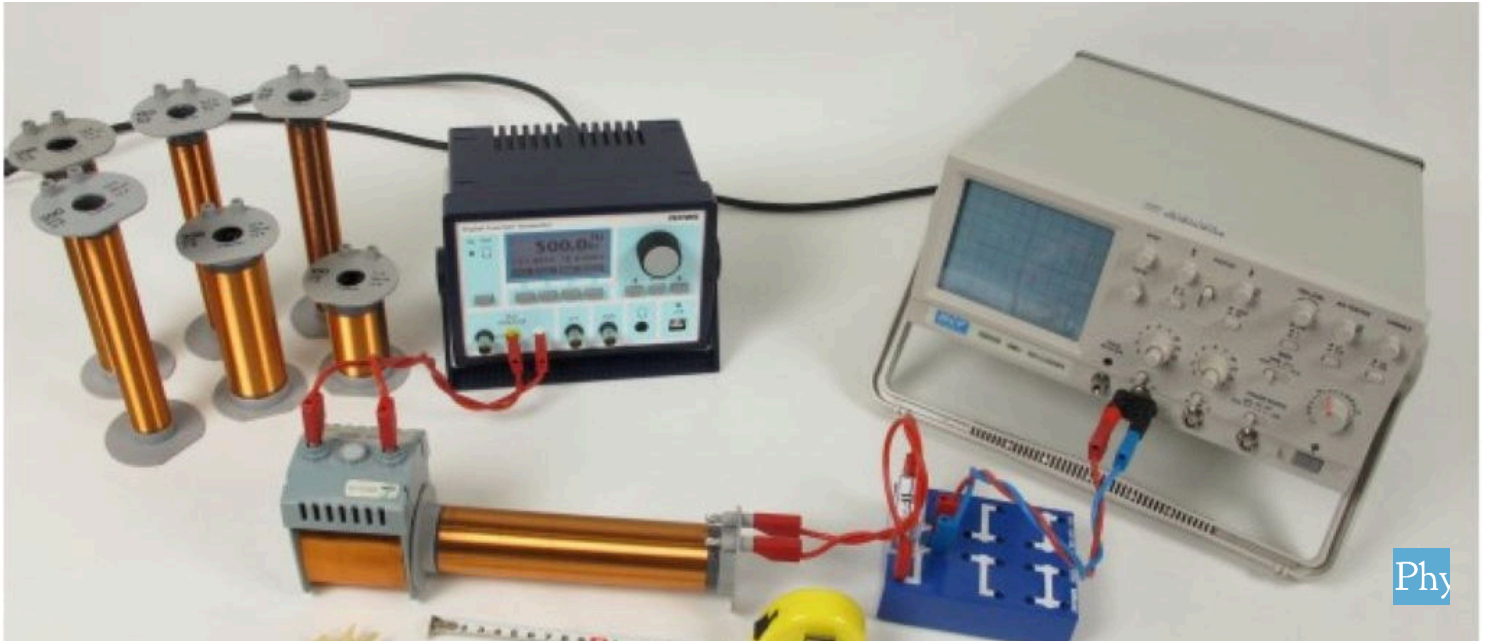


Inductance of solenoids



The goal of this experiment is to investigate the inductance of solenoids

Physics

Electricity & Magnetism

Magnetism & magnetic field

Applied Science

Engineering

Electrical Engineering

Properties of Electrical Devices



Difficulty level

hard



Group size

2



Preparation time

45+ minutes



Execution time

45+ minutes

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General information

Application

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Setup

When an alternating electric current flows through the wire of a solenoid, it will create a changing magnetic field within the solenoid. This changing magnetic field creates a changing magnetic flux that will induce an emf inside the loops of wire (known as inductance).

Other information (1/2)

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Prior knowledge



The students should have basic knowledge about electromagnetic resonance circuit.

Scientific principle



A square wave voltage of low frequency is applied to an oscillating circuit comprising coil and capacitor of known capacitance. The sudden change of voltage at both edges of the square wave signal induces a magnetic field in the primary coil which then couples into the solenoid and triggers a free damped oscillation in the secondary circuit. For different solenoids the natural frequencies of the circuits are measured and therewith the solenoids' inductances are calculated.

Other information (2/2)

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Learning objective



The goal of this experiment is to investigate the inductance of solenoids

Tasks



The natural frequency of the induced oscillation has to be measured each induction coil. From the natural frequency and the known capacitance calculate the inductances of the coils and determine the relationships between

1. inductance and number of turns,
2. inductance and solenoid length
3. as well as inductance and solenoid radius.

Theory (1/4)

If a current of strength I flows through a cylindrical coil (a.k.a. solenoid) of length l , cross sectional area $A = \pi \cdot r^2$, and number of turns N , a magnetic field is set up in the coil. When $l \gg r$ the magnetic field is uniform and the field strength H is given by

$$H = I \cdot \frac{N}{l} \quad (1)$$

The magnetic flux Φ is given by

$$\Phi = \mu_0 \cdot \mu \cdot H \cdot A \quad (2)$$

where μ_0 is the magnetic field constant and μ the absolute permeability of the surrounding medium. When this flux changes it induces a voltage between the ends of the coil,

$$U_{\text{ind}} = -N \cdot \frac{d\Phi}{dt} = -N \cdot \mu_0 \mu \cdot A \cdot \frac{dH}{dt} = -L \cdot \frac{dI}{dt} \quad (3)$$

Theory (2/4)

$$\text{where } L = \mu_0 \cdot \mu \cdot \pi \cdot \frac{N^2 \cdot r^2}{l} \quad (4)$$

is the coefficient of self-induction (inductance) of the coil. In practice the inductance of coils with $l > r$ can be calculated with greater accuracy by an approximation formula

$$L = 2.1 \cdot 10^{-6} \cdot N^2 \cdot r \cdot \left(\frac{r}{l}\right)^{3/4} \quad (5)$$

for $0 < \frac{r}{l} < 1$

In the experiment the inductance of various coils is calculated from the natural frequency of an oscillating circuit:

$$\omega_0 = \frac{1}{\sqrt{LC_{\text{tot}}}}$$

Theory (3/4)

$C_{\text{tot}} = C + C_i$ is the sum of the known capacitor and the input capacitance $C_i \approx 30 \text{ pF}$ of the oscilloscope, which exercises a damping effect on the oscillatory circuit and causes a negligible shift (approx. 1 %) in the resonance frequency.

The inductance is therefore represented by

$$L = (4\pi^2 v_0^2 C)^{-1} \quad (6)$$

where $v_0 = \frac{\omega_0}{2\pi} = \frac{1}{T \cdot 2\pi}$ is the natural frequency with T being the period of the oscillation.

Theory (4/4)

| Coil No. | N | 2r [mm] | l [mm] | L_{calc} [μH] |
|----------|-----|---------|--------|-------------------------------------|
| 1 | 300 | 41 | 160 | 830 |
| 2 | 300 | 33 | 160 | 568 |
| 3 | 300 | 26 | 160 | 374 |
| 4 | 200 | 41 | 105 | 506 |
| 5 | 150 | 41 | 50 | 221 |
| 6 | 100 | 26 | 160 | 93 |
| 7 | 75 | 26 | 160 | 23 |

Tab. 1: Coil data and calculated inductances (see eq. 5).

| Coil No. | T [μs] | ν [kHz] | L_{exp} [μH] | L_{calc} [μH] |
|----------|---------------------|-------------|------------------------------------|-------------------------------------|
| 1 | 130 | 7.7 | 911 | 830 |
| 2 | 100 | 10 | 539 | 568 |
| 3 | 80 | 12.5 | 345 | 374 |
| 4 | 100 | 10 | 539 | 506 |
| 5 | 64 | 15.6 | 221 | 221 |
| 6 | 40 | 25 | 86 | 93 |
| 7 | 20 | 50 | 22 | 23 |

Tab. 2: Measured natural frequencies and inductances from eqs. (5) and (6).

Equipment

| Position | Material | Item No. | Quantity |
|----------|--|------------|----------|
| 1 | PHYWE Digital Function Generator, USB | 13654-99 | 1 |
| 2 | Induction coils, 1 set (7 coils) | 11007-88 | 1 |
| 3 | Coil, 1200 turns | 06515-01 | 1 |
| 4 | Capacitor 470nF/250V, G1 | 39105-20 | 1 |
| 5 | Digital storage oscilloscope, 20 MHz, 2 channels, 100 MS/s | EAK-P-1335 | 1 |
| 6 | Adapter, BNC male/4 mm female pair | 07542-26 | 1 |
| 7 | Connection box | 06000-00 | 1 |
| 8 | Connecting cord, 32 A, 250 mm, red | 07360-01 | 1 |
| 9 | Connecting cord, 32 A, 250 mm, blue | 07360-04 | 1 |
| 10 | Connecting cord, 32 A, 500 mm, red | 07361-01 | 2 |
| 11 | Connecting cord, 32 A, 500 mm, blue | 07361-04 | 2 |



Setup and Procedure

Setup (1/2)

Perform the experimental set-up according to Figs. 1 and 2. Set the digital function generator to an amplitude of 20 V, frequency of 500 Hz or lower and to the square wave signal with signal-type out.

The solenoid has to be aligned carefully with the primary coil so that the magnetic field can couple efficiently from the primary coil into the solenoid.



Fig. 1: Experimental setup

Setup (2/2)


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The distance between the two coils should be maximized so that the effect of the excitation coil on the resonant frequency can be disregarded. there should be no iron components in the vicinity of the coils.

The tolerance of the oscilloscope time-base is given as 4 %. If a higher degree of accuracy is required, the time-base can be calibrated for all measuring ranges with the digital function generator prior to these experiments.

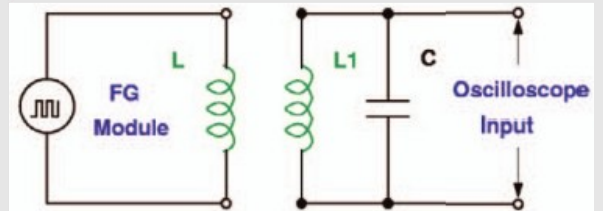


Fig. 2: Schematic circuit of the set-up. L is the primary coil with 1200 turns, L_1 labels the solenoid in the secondary circuit.

Procedure


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For each solenoid the settings of the oscilloscope have to be adjusted in such manner, that one damped oscillation can be fully seen on the screen and the peaks are clearly distinguishable as shown in Fig. 3.

The time between two peaks of the damped oscillation is the actual period length of the natural frequency and can be easily read from the screen by shifting the wave into appropriate positions. The lengths of the solenoids have to be measured with the measuring tape, the radius (i.e. the diameter) has to be determined with the vernier caliper and the numbers of turns are given.

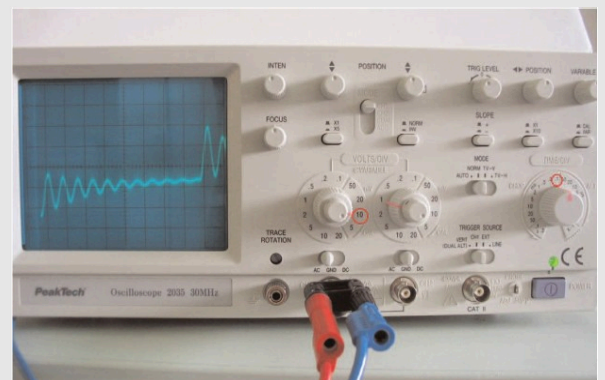


Fig. 3: Damped oscillation on the oscilloscope. The actual settings can vary for differing coils.



Evaluation

General (1/2)

In the following the evaluation of the obtained values is described with the help of example values. Your results may vary from those presented here.

From equation (5) we obtain the theoretical inductance values of the used coils. These are listed in table 1.

The following coils provide the relationships between inductance and radius, length and number of turns that we are investigating:

1. no. 3, 6, 7

2. no. 1, 4, 5

3. no. 1, 2, 3

As a difference in length also means a difference in the number of turns, the relationship between inductance and number of turns found in task 1 must also be used to solve task 2.

General (2/2)

In table 2 the measured natural frequencies are shown as well as the calculated values for L from relations (5) and (6) respectively. The experimental values L_{exp} are in good agreement with the theoretically expected values L_{calc} with a standard deviation of approximately 6%.

The coils with higher inductances show rather considerable deviations. As the inductance is proportional to the square of the oscillation's period length, this scatter is hard to reduce because especially for long-period oscillations the oscilloscope's accuracy is limited.

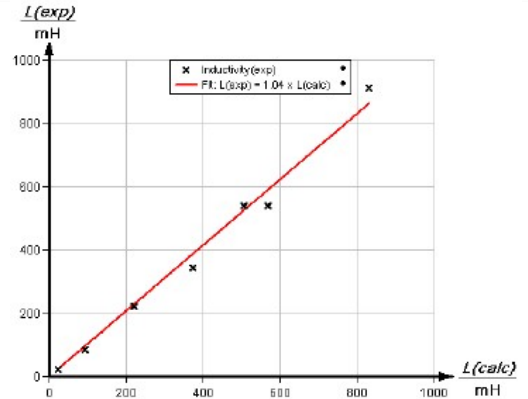


Fig. 4: Comparison of inductances L_{calc} with L_{exp} calculated from equations (5) and (6) respectively.

Task 1

To determine the relationship between inductance and number of turns consider coils with identical radius and length but different number of turns. The Coils no. 3, 6 and 7 meet these requirements.

In figure 5 the corresponding inductances are plotted in dependence of the number of turns.

Fitting the expression $L_{exp} = a \cdot n^b$ to the experimental values yields

$$b = 2.0003$$

which is in excellent agreement with the theoretical value $b_{theo} = 2$ (see eq. 5).

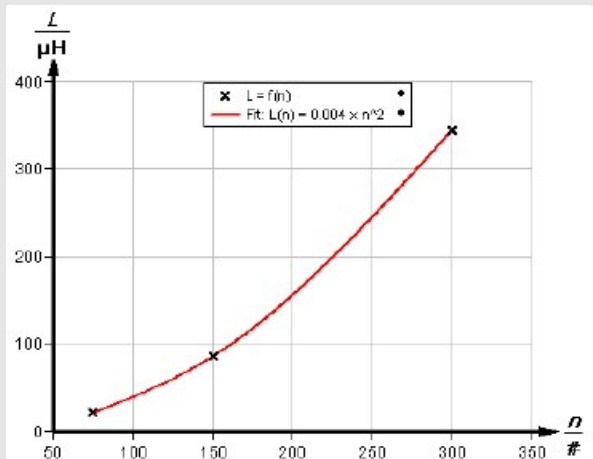


Fig. 5: Relation between inductance and number of turns.

Task 2

To determine the relationship between inductance and length of coil consider coils with identical radius but different lengths. The Coils no. 1, 4 and 5 meet these requirements. As the relation between inductance and number of turns is already known, the inductances can be normalized by the number of turns. Therefore, consider the relationship between inductance normalized by turn number squared and the length of coil. In figure 6 the corresponding values are plotted in dependence of the coil length.

Fitting the expression $L_{\text{exp}}/n^2 = a \cdot l^b$ to the experimental values yields $b = -0.67$ which is in fair agreement with the theoretical value $b_{\text{theo}} = -0.75$ (see eq. 5).

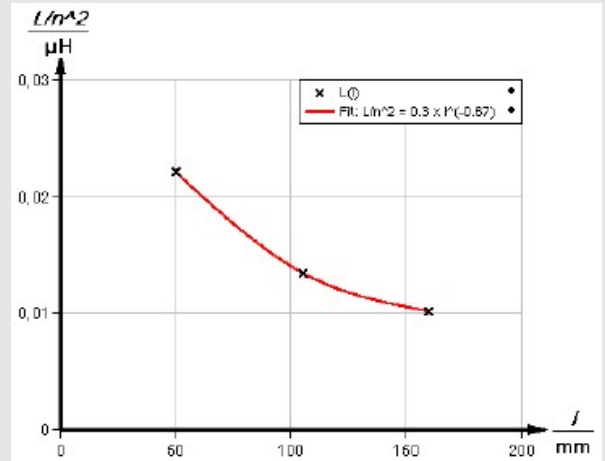


Fig. 6: Relation between inductance and length of coil.

Task 3

To determine the relationship between inductance and radius of coil consider coils with identical lengths but different radii. The Coils no. 1, 2 and 3 meet these requirements. As the relation between inductance and number of turns is already known, the inductances can be normalized by the number of turns. Therefore, consider the relationship between inductance normalized by turn number squared and the radius of the coils. In figure 7 the corresponding values are plotted in dependence of the coil radius.

Fitting the expression $L_{\text{exp}}/n^2 = a \cdot r^b$ to the experimental values yields $b = 1.82$ which is in good agreement with the theoretical value $b_{\text{theo}} = 1.75$ (see eq. 5).

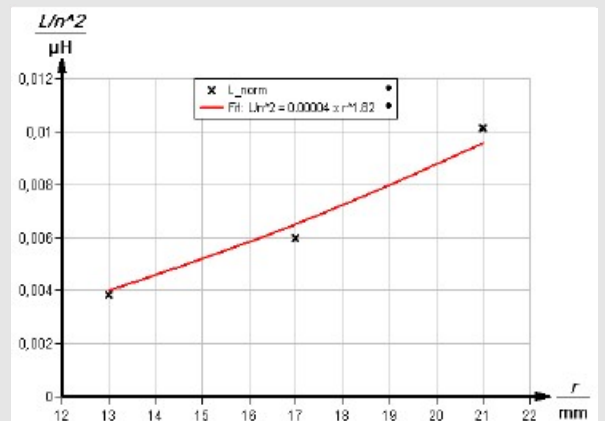


Fig. 7: Relation between inductance and radius of the coil.