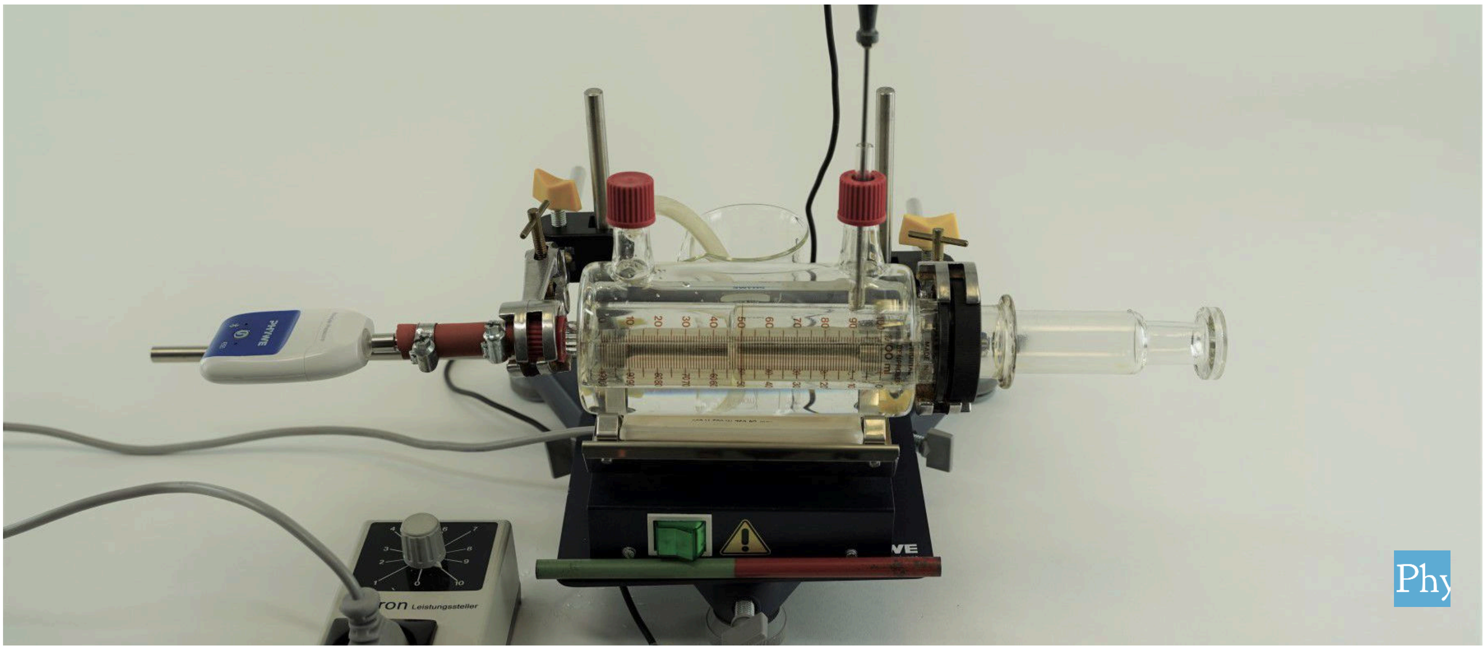
Equation of state for ideal gases (gas laws: Gay-Lussac, Amontons, Boyle)



Physics	Thermodynamics	Kinetic gas theory & gas laws
Chemistry	General Chemistry	Stoichiometry
Chemistry	Physical chemistry	Gas laws

Difficulty level	Group size	Preparation time	Execution time
medium	1	20 minutes	45+ minutes

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Teacher information

Application

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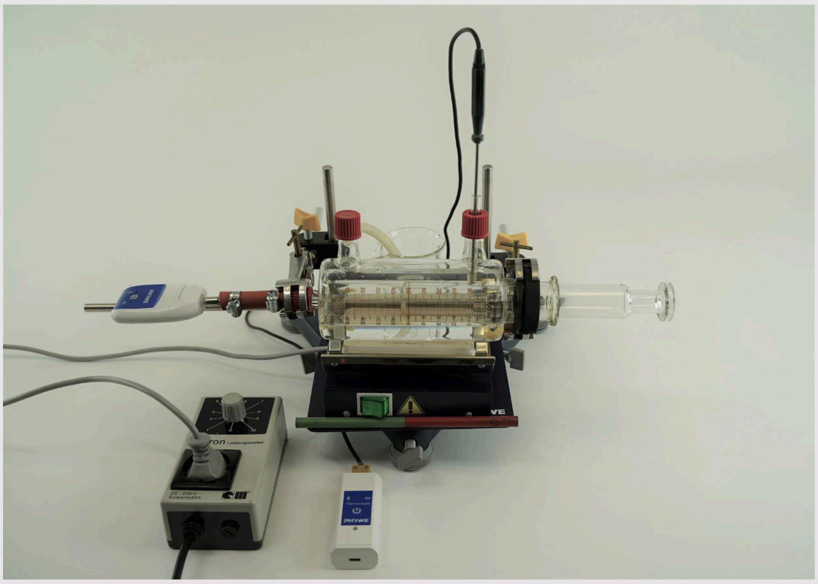

This experimental setup combines the experimentation of the three gas laws postulated by Robert Boyle, Jacques Charles, Amadeo Avogadro, Guillaume Amontons and Joseph-Louis Gay-Lussac. These three experiments described define the modern laws of thermodynamics.

The ideal gas law describes an equation containing pressure P, temperature T, volume V, amount of substance n, particle count N and mass m.

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Other teacher information (1/2)

Prior knowledge

The students have to be familiar with units like pressure, temperature, mass, volume and can perform calculations with them. In addition, they have to be familiar with general good laboratory practice and general laboratory safety regulations.

Scientific principle

The state of a gas is determined by temperature, pressure and amount of substance. For the limiting case of ideal gases, these state variables are linked via the ideal gas law. For a change of state under isobaric conditions this equation converts to Gay-Lussac's first law while under isochoric conditions it becomes Amontons' and in the case of isothermal process control it converts to Boyle and Mariotte's law.

Other teacher information (2/2)

Learning objective

In this experiment, the students are getting familiar with different behaviours of gases and deepen their knowledge in physical equations. By going through the different parts with the same volume of air while changing the external influences, they get to know the correlation of pressure, temperatur and volume. It is a simple introduction in thermodynamics.

Tasks

1. Experimentally investigate the validity of the three gas laws for a constant amount of gas (air).
2. Calculate the universal gas constant from the relationship obtained.
3. Calculate the thermal coefficient of expansion from the results of measurements under isobaric conditions.
4. Calculate the thermal coefficient of tension from the results of measurements under isochoric conditions.

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Theory

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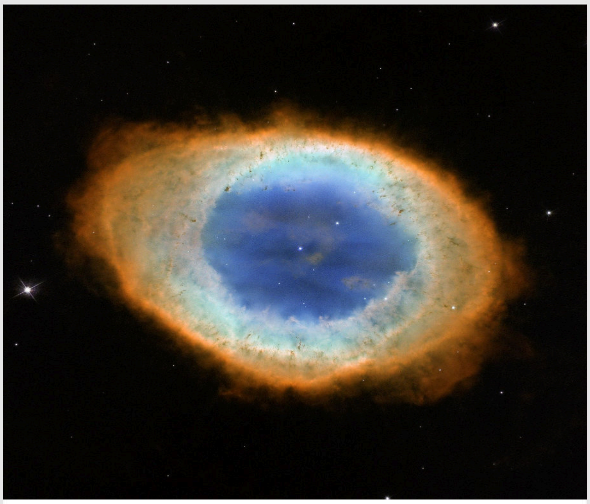

Fig. 1: Gases are present on every planet in the universe

These three experiments demonstrate the ideal gas law. Despite multiple limitations, this law is a good approximation of the behavior of many gases at many given conditions.
The ideal gas law describes an equation containing pressure P, temperature T, volume V, amount of substance n, particle count N and mass m. It is defined as

$$P \cdot V = n \cdot R \cdot T$$

with the Avogadro or ideal gas constant R.

Safety instructions

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- When handling chemicals, you should wear suitable protective gloves, safety goggles, and suitable clothing.
- For this experiment the general instructions for safe experimentation in science lessons apply.
- For H- and P-phrases please consult the safety data sheet of the respective chemical.

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Setup and Procedure

Tasks

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Equipment

Position	Material	Item No.	Quantity
1	Set gas laws with glass jacket, 230 V	43003-88	1
2	Cobra SMARTsense - Thermocouple, -200 ... +1200 °C (Bluetooth + USB)	12938-01	1
3	Immersion probe NiCr-Ni, steel, -50...400 °C	13615-03	1
4	Cobra SMARTsense - Absolute Pressure, 20 ... 400 kPa (Bluetooth + USB)	12905-01	1
5	measureLAB, multi-user license	14580-61	1
6	Power regulator, 230 V, with phase controlled modulator	32286-93	1
7	Glass tube, straight, l=80 mm, 10/pkg.	36701-65	1
8	Hose clamp for 8-12 mm diameter	41000-00	2
9	Silicone tubing i.d. 7mm, 1 m	39296-00	1
10	Support rod, stainless steel, l = 250 mm, d = 10 mm	02031-00	1
11	Support rod, stainless steel, 500 mm	02032-00	1
12	Boss head	02043-00	6
13	Adapter Luer Lock male, to tubing, d= 8 mm for Cobra SMARTsense Absolute Pressure	39284-00	1
14	Holder for Cobra SMARTsense	12960-00	2

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Additional equipment

Position	Material	Quantity
1	PC with Windows XP® or higher	1

Setup (1/3)

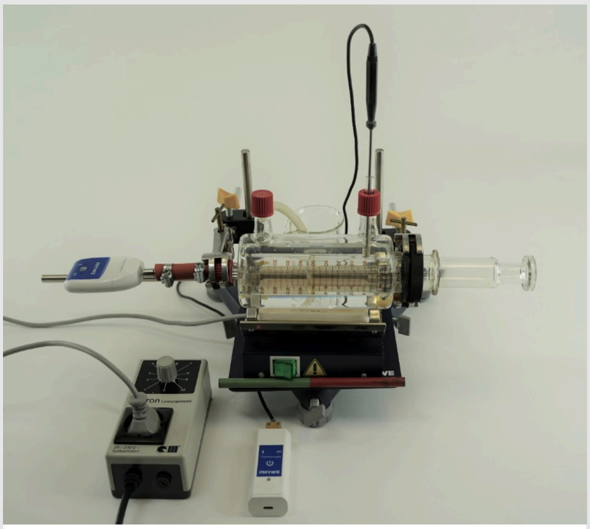

Fig. 2: Assembled experiment

- Set up the experiment as shown in Fig. 1.
- Connect the Cobra SMARTsense Thermocouple with the temperature probe.
- Turn on the Cobra SMARTsense Thermocouple and connect it with the PC.
- Start the software “measureLAB” on your computer and choose the experiment from the start screen ("PHYWE experiments", search for "P2320162", and click on the folder that contains this experiment). All necessary presettings will be loaded.
- The Cobra SMARTsense Thermocouple sensor is automatically recognized.

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Setup (2/3)

- Install the gas syringe in the glass jacket as described in the operating instructions supplied with the glass jacket. Pay particular attention to the air-tightness.
- As an exception here, because no air is to be allowed to leak out even at higher pressures, lubricate the plunger with a few drops of multigrade motor oil, so that the glass plunger is covered with an uninterrupted clear film of oil throughout the entire experiment; but avoid excess oil.
- Fill the glass jacket with water via the funnel and insert a magnetic stirrer bar.
- Connect a silicone tube to the hose nipple of the jacket's upper tubular sleeve so that the bath fluid which expands on heating can flow through the tube into a beaker.
- Insert the thermocouple and place it as close to the syringe as possible.

Setup (3/3)

- After adjusting the initial volume of the gas syringe to exactly 50 ml, connect the nozzle of the gas syringe to Cobra SMARTsense Absolute Pressure via a short piece of rubber tubing. Keep the tubing connections as short as possible.
- Secure the tubing on both the gas syringe's nozzle and on the reducing adapter with hose clips.
- Before starting, you need to create a calculated channel for the Volume with the expected maximum of 65, the expected minimum of 50 and the calculationterm V = index + 50 by clicking on $\sqrt{\alpha}$ and +.

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Procedure (1/6) - Boyle and Mariotte's law

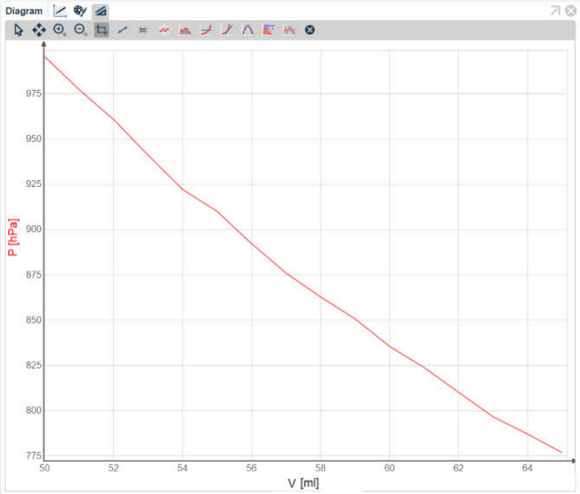

Fig. 3: Correlation between the volume V and the pressure p at constant temperatures

- Start the measurement with
- Subsequently expand the enclosed quantity of air in 1 ml steps to a volume of approximately 65 ml.
- Record the volume for each step by clicking on
- Mix the water in the glass jacket by moving the magnetic stirrer bar with the aid of a bar magnet and facilitate pressure equilibration in the gas syringe by turning the plunger.

Procedure (2/6) - Boyle and Mariotte's law

- Terminate the measurement by pressing
- After termination, the measureLAB software presents a graph that shows the correlation between volume and pressure as constant temperature.
- To have the plot of pressure versus the reciprocal volume, click on the symbol to open the data pool.
- Now you can perform some channel modifications by clicking on $\sqrt{\alpha}$. First, drag and drop your measure data (volume) to measurements, then drag and drop the data into your formula.
- Go back to data pool and select the measure data for pressure and your modified channel pVT. When selected, choose the option 'Diagram' and the software will present you the desired graph that shows the correlation between pressure p and the quantity 1/V.
- With that you can let the program show the slope.

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Procedure (3/6) - Gay-Lussac’s law

- Start the measurement with
- Record the first value for the initial temperature by clicking
- Switch on the heating apparatus and adjust the power regulator so that the glass jacket is slowly heated.
- Mix the water in the glass jacket by moving the magnetic stirrer bar with the aid of a bar magnet and facilitate pressure equilibration in the gas syringe by turning the plunger.
- After each 1 ml increase in volume, take the next value.

Procedure (4/6) - Gay-Lussac’s law

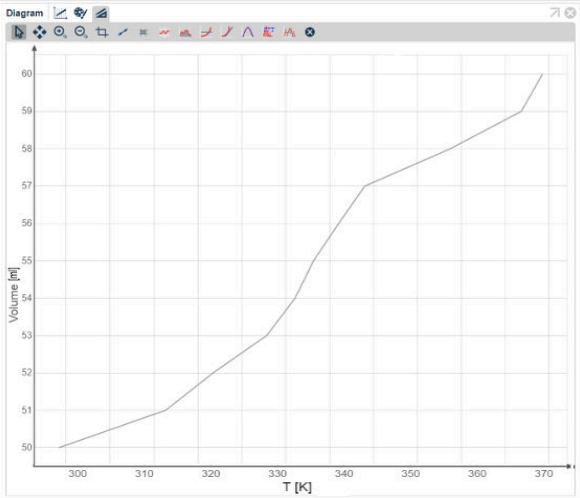

Fig. 4: Dependence of the volume V on the temperature T at constant pressure

- After the gas volume has reached 60 ml, switch off the heating apparatus and terminate the measurement by pressing
- To have the plot of the quantity pV/T versus volume, go to data pool and click on
- Now you can perform some channel modifications, drag and drop the measure data for volume, temperature and pressure to measurements. Subsequently, drag and drop the data to the formula
- Go to data pool and select the measure data for volume and your modified channel pVT. When selected, choose the option 'Diagram' and the software will present you the desired graph.

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Procedure (5/6) - Amontons’ law

- Start the measurement with
- Subsequently, adjust the heating apparatus to slow heating with the power regulator.
- Mix the water in the glass jacket by moving the magnetic stirrer bar with the aid of a bar magnet and facilitate pressure equilibration in the gas syringe by turning the plunger.
- Record the pressure corresponding to the initial temperature by clicking on
- After each temperature increase of 5 K, push the plunger rapidly into the gas syringe until the gas volume is compressed to the initial volume of V = 50 ml and take the next value by clicking on
- After the temperature has reached approximately 370 K or if there is an evident loss of air during compression, switch off the heating apparatus and terminate the measurement by pressing

Procedure (6/6) - Amontons’ law

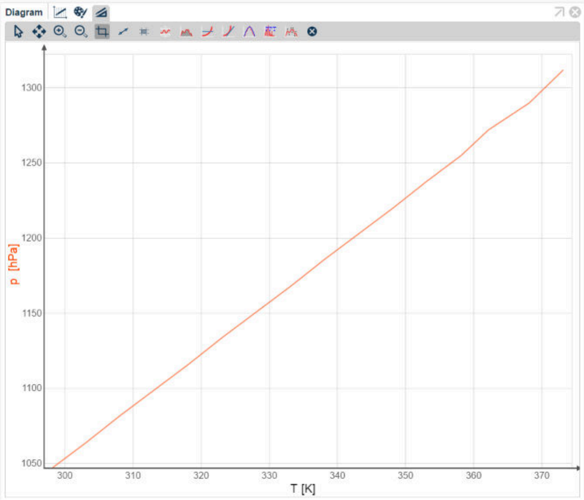

Fig. 5: Dependence of the pressure p on the temperature T at constant volume

- The figure on the left side shows the graph for the dependence of the pressure p on the temperature at constant volume as it is then presented by the program.
- To have the plot of the quantity pV/T versus temperature, go to data pool and click on $\sqrt{\alpha}$
- Now you can perform some channel modifications, drag and drop the measure data for volume, temperature and pressure to measurements. Subsequently, drag and drop the data to the formula.
- Go to data pool and select the measure data for temperature and your modified channel pVT. When selected, choose the option 'Diagram' and the software will present you the desired graph.

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Evaluation (1/12)

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The state of a gas is a function of the state variables temperature T, pressure p and the amount of substance n, which reciprocally determine one another. Thus, the dependence of pressure on the temperature, volume and amount of substance variables is described by the total differential.

$$dV = (\frac{\delta V}{\delta T})_{p,n} dT + (\frac{\delta V}{s\delta p})_{T,n} dp + (\frac{\delta V}{\delta n})_{T,V} dn \qquad (1.1)$$

Analogously, the following is true for the change of pressure with *T*, *V* and *n*:

$$dp = (\frac{\delta p}{\delta T})_{V,n} dT + (\frac{\delta p}{s\delta V})_{T,n} dV + (\frac{\delta p}{\delta n})_{T,V} dn \qquad (1.2)$$

Evaluation (2/12)

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This relationship simplifies for a given amount of substance (n = const., dn = *0*; enclosed quantity of gas in the gas syringe) and isothermal change of state (T = const., dT = *0*) to

$$dV = (\frac{\delta V}{\delta T})_{T,n} dp \qquad (2.1)$$

and

$$dp = (\frac{\delta p}{\delta T})_{V,n} dT \qquad (2.2)$$

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Evaluation (3/12)

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The partial differential quotient $(\delta V/\delta p)T,n$ resp. $(\delta p/\delta V)T,n$ corresponds geometrically to the slope of a tangent to the function $V = f(p)$ or $p = f(V)$ and therefore characterises the mutual dependence of pressure and volume. The degree of this dependence is determined by the initial volume or the initial pressure. One thus defines the cubic compressibility coefficient by referring it to V or V_0 at $T_0 = 273.15K$

$$X_0 = \frac{1}{V_0}(\frac{\delta V}{\delta p})_{T,n} \quad (3)$$

The partial differential quotient $(\delta p/\delta T)V,n$ corresponds geometrically to the slope of a tangent to the function $p = f(T)$ and thus characterises the dependence of the pressure on the temperature. The degree of this dependence is determined by the initial pressure. Therefore, one defines the thermal coefficient of tension β_0 as a measure of the temperature dependence by referring it to p or p_0 at $T_0 = 273.15K$

$$\beta_0 = \frac{1}{V_0}\frac{\delta 0}{\delta T}_{p,n} \quad (4)$$

Evaluation (4/12)

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The partial differential quotient $(\delta V/\delta T)p,n$ corresponds geometrically to the slope of a tangent to the function $V = f(T)$ and thus characterises the mutual dependence of volume and temperature. The degree of this dependence is determined by the initial volume. The thermal coefficient y_0 of expansion is therefore defined as a measure of the temperature dependence of the volume by referring it to V or V_0 at $T_0 = 273.15K$

$$y_0 = \frac{1}{V_0}(\frac{\delta V}{\delta T})_{p,n} \quad (5)$$

For the limiting case of an ideal gas (sufficiently low pressures, sufficiently high temperatures), the correspondence between the state variables p, V, T and n is described by the ideal gas law:

$$pV = nRT \quad (6)$$

with R as the Universal gas constant.

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Evaluation (5/12)

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For cases of constant quantity of substances and isothermal process control this equation changes into the following equations:

pV = const. (6.1) and p = const. (6.2)

According to this correlation, which was determined empirically by Boyle and Mariotte, a pressure increase is accompanied by a volume decrease and vice versa. The graphic representation of the functions $V = f(p)$ or $p = f(V)$ results in hyperbolas. In contrast, plotting the pressure p against the reciprocal volume $1/V$ results in straight lines where at $p = 0$ at $1/V$. From the slope of these linear relationships,

$$(\frac{\delta p}{\delta v^{-1}})_{T,n} = nRT \quad (7)$$

it is possible to determine the gas constant R experimentally when the enclosed constant quantity of air n is known. This is equal to the quotient of the volume V and the molar volume V_m, $n = \frac{V}{V_m}$ (8) which is $V_0 = 22.414 l \cdot mol^{-1}$ at $T_0 = 273.15 K$ and $p_0 = 1013.25 hPa$ at standard conditions.

Evaluation (6/12)

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A volume measured at p and T is therefore first reduced to these conditions using the relationship obtained from (6):

$$\frac{p_0 V_0}{T_0} = \frac{p_1 V_1}{T_1} = \frac{pV}{T} \quad (9)$$

For the limiting case of an ideal gas (sufficiently low pressure, sufficiently high temperature), the integration of a differential equation resulting from (1.2) and (4), where β_0 = const., yields

$$\frac{p_0}{T_0} = \frac{p}{T} \quad (10.1)$$

and

$$p = const. T$$

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Evaluation (7/12)

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According to this correlation, which was discovered by Charles and Amontons, the graphic presentation of the pressure as a function of the temperature results in an ascending straight line where $p=0$ at $T=0$.

From (4) and the ideal gas law (6) the following is true for the slope of these linear relationships

$$(\frac{\delta p}{\delta T})_{V,n} = p_0\beta_0 = \frac{nR}{V} \quad (11)$$

From this, the thermal coefficient of tension β_0 and the universal gas constant R can be determined for a known initial pressure p_0 and a known quantity of substance n. The enclosed constant amount of substance n is equal to the quotient of the volume V and the molar volume V_m.

For the limiting case of an ideal gas (sufficiently low pressure, sufficiently high temperature), the integration of a differential equation resulting from (1.2) and (5), where y_0 = constant, yields

$$\frac{V_0}{T_0} = \frac{V}{T} \quad (12.1) \quad \text{and } V = const. T \quad (12.2)$$

Evaluation (8/12)

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According to this correlation, which was discovered by Gay-Lussac, the graphic presentation of the volume as a function of the temperature provides a scending straight lines where $V=0$ for $T=0$. From (5) and the ideal gas law (6) the following is true for the slope of these linear relationships:

$$(\frac{\delta V}{\delta T}) = V_0\gamma_0 = \frac{nR}{p} \quad (13)$$

From this, the thermal coefficient of expansion γ_0 and the universal gas constant R are experimentally accessible for a known initial volume V_0 and a known amount of substance n.

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Evaluation (9/12)

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Data and results

The teoretical values for an ideal gas are

$R(lit.) = 8.31441 Nm \cdot K^{-1} \cdot mol^{-1} = J \cdot K^{-1} \cdot mol^{-1}$

$\gamma_0(lit.) = 3.661 \cdot 10^{-3} K^{-1}$

$\beta_0(lit.) = 3.661 \cdot 10^{-3} K^{-1}$

Evaluation (10/12)

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1. Boyle and Mariotte's law

The data created during the first experiment confirms the validity of Boyle and Mariotte's law. From the slope obtained for $n = 2.086 mmol$ and $T = 295.15 K$, $(\delta p / \delta V^{-1})_{T,n} = 4.6464 kPa/m^{-3} = 4.6464 Nm$ of the linearised corelation between p and $1/V$, the universal gas constant can be calculated to be $R = 7.547 Nm \cdot K^{-1}$.

The deviation from the literature value is due to the unavoidable lack of gas-tightness with increasing deviation from atmospheric pressure through compression or expansion, whereby the condition $dn = 0$ is violated and the observed slope $(\delta p / \delta V^{-1}) T$ is diminished in comparison with the value measurable with a constant quantity of substance.

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Evaluation (11/12)

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2. Gay-Lussac's law

The investigation of the correlation between volume and temperature with a constant quantity of gas of $n = 2.23 mmol$, calculated according to the relations (8) and (9), confirms the validity of the Gay-Lussac's first law, with a linear relationship.

From the corresponding slope $(\delta V/\delta T)_{p,n} = 0.18 ml/K$ and for the initial volume $V_0 = 50 ml$, the following values are obtained for the universal gas constant R and the coefficient of thermal expansion γ_0.

$R(exp.) = 8.07174 Nm \cdot K^{-1} \cdot mol^{-1}$

$\gamma_0(exp.) = 3.04 \cdot 10^{-3} K^{-1}$

Evaluation (12/12)

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3. Amontons' law

The investigation of the correlation between pressure and temperature with a constant quantity of gas of , calculated according to the relations (8) and (9), confirms the validity of the Charles' (Amontons') law with the linear relationship demonstrated in the third experiment.

From the corresponding slope $(\delta p/\delta T)_{V,n} = 3.72 hPa/K$ and for the initial pressure $p_0 = 1002.2 hPa$, the following values are obtained for the universal gas constant $R$and the coefficient of thermal tension β_0.

$R(exp.) = 8.34 Nm \cdot K^{-1} \cdot mol^{-1}$

$\beta_0(exp.) = 3.71 \cdot 10^{-3} K^{-1}$

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