Diffraction at a slit and Heisenberg's uncertainty principle (Item No.: P2230101)

Curricular Relevance

Area of Expertise: Physics

Education Level: University

Topic: Light and Optics

Subtopic: Diffraction and Interference

Experiment: Diffraction at a slit and Heisenberg's uncertainty principle

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	10 Minutes	20 Minutes	2 Students

Additional Requirements:

- PC/Laptop with at least Windows XP

Experiment Variations:

Keywords:

Diffraction, Diffraction uncertainty, Kirchhoff's diffraction formula, Measurement accuracy, Uncertainty of location, Uncertainty of momentum, Wave-particle dualism, De Broglie relationship

Introduction

Overview

The distribution of intensity in the Fraunhofer diffraction pattern of a slit is measured. The results are evaluated both from the wave pattern viewpoint, by comparison with Kirchhoff's diffraction formula, and from the quantum mechanics standpoint to confirm Heisenberg's uncertainty principle.

Fig. 1: Experimental set-up to investigate the diffraction intensity of a single slit.

Equipment

Position No.	Material	Order No.	Quantity
1	Diaphragm, 3 single slits	08522-00	1
2	Object holder, 5x5 cm	08041-01	1
3	Digital array camera	35612-99	1
4	Optical bench expert, l = 1500 mm	08281-00	1
5	Base for optical bench expert, adjustable	08284-00	2
6	Slide mount for optical bench expert, h = 30 mm	08286-01	3
7	Diodelaser, red, 1 mW, 635 nm	08761-99	1
8	Screen, white, 150x150 mm	09826-00	1

Tasks

1. To measure the intensity distribution of the Fraunhofer diffraction pattern of a single slit (e.g. 0.1 mm). The heights of the maxima and the positions of the maxima and minima are calculated according to Kirchhoff's diffraction formula and compared with the measured values.
2. To calculate the uncertainty of momentum from the diffraction patterns of single slits of differing widths and to confirm Heisenberg's uncertainty principle.

Set-up and procedure

The diaphragm with the different single slits is set in the object holder and placed in front of the laser beam at a distance of around 5 mm, as shown in Fig. 1. Slide the diaphragm along the object holder to alternate between the different slit widths. The distribution of the intensity in the diffraction pattern is measured with the digital array camera as far behind the slit as possible. Connect the camera to the USB port of the PC, and the software should install automatically. For further information click the "Help" button in the bottom right. The measurements should be taken in a darkened room or in constant natural light.

Caution: Never look directly into a non attenuated laser beam

The principal maximum, and the first secondary maximum on one side, of the symmetrical diffraction pattern of a slit 0.1 mm wide (for example) are recorded. For the other slits, it is sufficient to record the two minima to the right and left of the principal maximum, in order to determine α (Fig. 2). For each slit, measure a (distance between intensity maximum and minimum points for central peak) and b (distance between slit and screen/camera) shown in Fig.2. Calculate and compare results with Table 1.

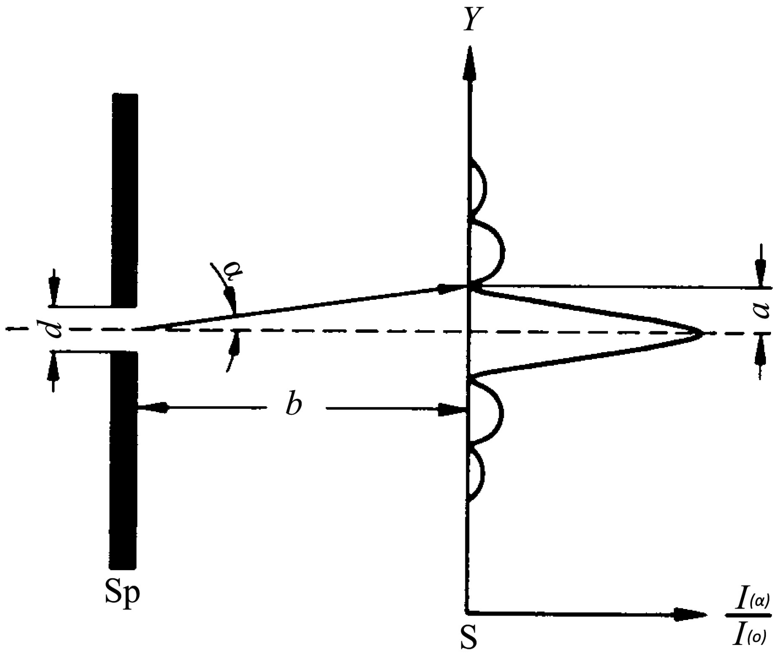

Fig. 2: Diffraction (Fraunhofer) at great distance (Sp = aperture or slit; S = screen).

Theory and evaluation

1. Observation from the wave pattern point

When a parallel, monochromatic and coherent light beam of wavelength λ passes through a single slit of width d, a diffraction pattern with a principal maximum and several secondary maxima appears on the screen (Fig. 2). The intensity I, as a function of the angle of deviation α, in accordance with Kirchhoff's diffraction formula, is

$$I(\alpha) = I(0) \cdot \left(\frac{sin(ß)}{ß}\right)^2 \quad (1)$$

where

$$ß = \frac{\pi d}{\lambda} \cdot sin(\alpha).$$

The intensity minima are at

$$\alpha_n = arcsin\, n \cdot \frac{\lambda}{d}$$

where $n = 1, 2, 3...$

The angle for the intensity maxima are

$$\alpha_0' = 0$$
$$\alpha_1' = arcsin\, 1.430 \cdot \frac{\lambda}{d}$$
$$\alpha_2' = arcsin\, 2.459 \cdot \frac{\lambda}{d}$$

The relative heights of the secondary maxima are:

$$I(\alpha_1) = 0.0472 \cdot I(0)$$
$$I(\alpha_2) = 0.0165 \cdot I(0)$$

The measured values (Fig. 3) are compared with those calculated.

Minima

Measurement	Calculation
$\alpha_1 = 0.36^\circ$	$\alpha_1 = 0.36^\circ$
$\alpha_2 = 0.72^\circ$	$\alpha_2 = 0.72^\circ$
$\alpha_3 = 1.04^\circ$	$\alpha_3 = 1.07^\circ$

Maxima

Measurement	Calculation
$\alpha_1 = 0.52^\circ$	$\alpha_1 = 0.51^\circ$
$\alpha_2 = 0.88^\circ$	$\alpha_2 = 0.88^\circ$
$\frac{I(\alpha_1)}{I_0} = 0.044$	$\frac{I(\alpha_1)}{I_0} = 0.047$
$\frac{I(\alpha_2)}{I_0} = 0.014$	$\frac{I(\alpha_2)}{I_0} = 0.017$

Kirchhoff's diffraction formula is thus confirmed within the limits of error.

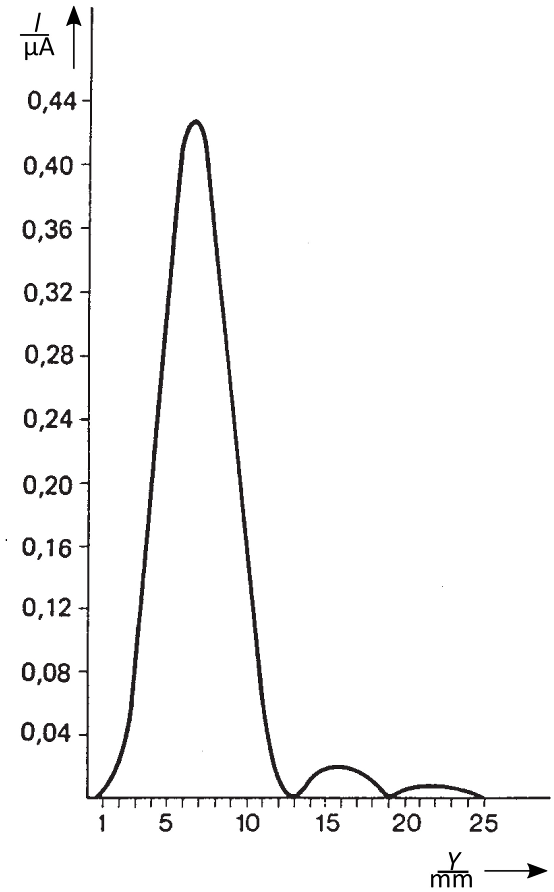

Fig. 3: Intensity in the diffraction pattern of a 0.1 mm wide slit at a distance of 1140 mm. The photocurrent is plotted as a function of the position.

2. Quantum mechanics treatment

The Heisenberg uncertainty principle states that two canonically conjugate quantities such as position and momentum cannot be determined accurately at the same time. Let us consider, for example, a totality of photons whose residence probability is described by the function f_y and whose momentum by the function f_p. The uncertainty of location y and of momentum p are defined by the standard deviations as follows:

$\Delta y \cdot \Delta p \geq \frac{h}{4\pi},$ (2)

where $h = 6.6262 \cdot 10^{-34}\,\mathrm{Js}$, Planck's constant (“constant of action”), the equals sign applying to variables with a Gaussian distribution. For a photon train passing through a slit of width d, the expression is

$\Delta y = d.$ (3)

The photons in front of the slit move only in the direction perpendicular to the plane of the slit (x-direction), and after passing through the slit they also have a component in the y-direction.

The probability density for the velocity component V_y is given by the intensity distribution in the diffraction pattern. We use the first minimum to define the uncertainty of velocity (Fig. 2 and Fig. 4)

$\Delta v_y = c \cdot sin\, \alpha_1$ (4)

where α_1 = angle of the first minimum.

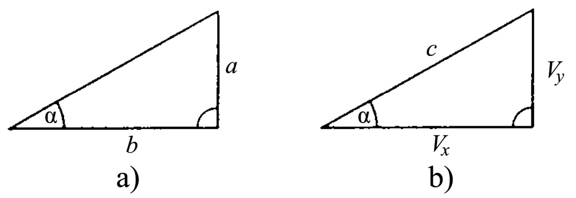

Fig. 4: Geometry of diffraction at a single slit. a) path covered and b) velocity component of a photon.

The uncertainty of momentum is therefore

$$\Delta p_y = m \cdot c \cdot sin\ \alpha_1 \quad (5)$$

where m is the mass of the photon and c is the velocity of light. The momentum and wavelength of a particle are linked through the *de Broglie relationship*:

$$\frac{h}{\lambda} = p = m \cdot c. \quad (6)$$

Thus,

$$\Delta p_y \frac{\lambda}{h} = sin\ \alpha_1\,. \quad (7)$$

The angle α_1 of the first minimum is thus

$$sin\ \alpha_1 = \frac{\lambda}{d}, \quad (8)$$

according to (1). If we substitute (8) in (7) and (3) we obtain the uncertainty relationship

$$\Delta y \cdot \Delta p_y = h. \quad (9)$$

If the slit width Δy is smaller, the first minimum of the diffraction pattern occurs at larger angles α_1.

In our experiment the angle α_1 is obtained from the position of the first minimum (Fig. 4a):

$$tan\ \alpha_1 = \frac{a}{b}. \quad (10)$$

If we substitute (10) in (7) we obtain

$$\Delta p_y = \frac{h}{\lambda} sin\left(arctan\frac{a}{b}\right). \quad (11)$$

Substituting (3) and (11) in (9) gives

$$\frac{d}{\lambda} sin\left(arctan\frac{a}{b}\right) = 1, \quad (12)$$

after dividing by h.

The results of the measurements in Table 1 confirm (12) within the limits of error.

Table 1: Measurement results.

Width of slit*	First minimum		$\frac{d}{\lambda} sin\left(arctan\frac{a}{b}\right)$
d/mm	a/mm	b/mm	
0.101	7.25	1140	1.01
0.202	3.25	1031	1.01
0.051	10.8	830	1.05

* The widths of the slits were measured under the microscope. Rounded values for d are sufficient for calculation.