Moments of inertia and torsional vibrations with CobraSMARTsense

Physics | Mechanics | Circular motion & rotation

Difficulty level	Group size	Preparation time	Execution time
hard	1	20 minutes	40 minutes

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General information

Application

Experimental setup

The moment of inertia $\hat{I}$ (also commonly $\hat{J}$ or $\hat{\Theta}$) in the case of rotational movements is the counterpart to mass m in linear movements. Together with the angular velocity $\vec{\omega}$, it determines how large the angular momentum $\vec{L}$ or rotational energy E_{rot} of a body is in a rotational movement.

The moment of inertia can be calculated for a body from the volume integral if its density distribution $\rho(\vec{r})$ is known:

$$I = \int_V \vec{r}_\perp^{\,2} \rho(\vec{r}) \mathrm{d}V$$

$\vec{r}_\perp$ corresponds to the component of $\vec{r}$ that is perpendicular to the axis of rotation $\vec{\omega}$.

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Other information (1/2)

Prior knowledge

Basic knowledge of physical quantities such as momentum, mass and speed as well as the theoretical description of a harmonic oscillation (for example in a mathematical pendulum) should be available. Ideally, terms like angular momentum, moment of inertia and rotational velocity should already be worked out theoretically.

Scientific principle

In the case of rotational movements, different parts of a rigid body move at different velocities. For the mathematical description of a rotational movement, therefore, instead of the mass of the body, its so-called moment of inertia (the inertia tensor) is used as a kind of measure of the density distribution around the axis of rotation.

Other information (2/2)

Learning objective

After the successful completion of this experiment you will be able to theoretically calculate the moment of inertia of different rigid bodies and also to determine it experimentally from the period of a torsional vibration. You will also gain knowledge about Steiner's theorem.

Tasks

The following will be determined:

1. The angular restoring moment of the spiral spring.
2. The moment of inertia of different ridgid bodies (two discs, two cylinders, a sphere)
3. The moment of inertia of two point masses, as a function of the perpendicular distance to the axis of rotation with its centre of gravity in the axis of rotation.

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Safety instructions

The general instructions for safe experimentation in science lessons apply to this experiment.

Theory (1/4)

The relationship between the angular momentum $\vec{L}$ of a rigid body in a stationary coordinate system with its origin at the center of gravity, and the moment $\vec{T}$ acting on it, is given by:

$$\vec{T} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{L}$$

Whereas the angular momentum is expressed by the angular velocity $\vec{\omega}$ and the inertia tensor $\hat{I}$:

$$\vec{L} = \hat{I} \otimes \vec{\omega}$$

For the given rotation around the z-axis of this experiment, the equation for the angular momentum reduces to the z-component L_Z and only depends on the z-component of the moment of inertia I_Z:

$$L_Z = I_Z \cdot \omega$$

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Theory (2/4)

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Insertion into the first equation leads to:

$$T_Z = I_Z \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} = I_Z \cdot \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}$$

with φ being the angle of rotation around the z-axis. On the other hand, according to Hooke's law, the moment T_Z that is necessary to deflect to a spiral spring to a certain angle is defined as

$$T_Z = -D \cdot \varphi \tag{1}$$

with D being the angular restoring constant of the spring. Comparison of both above equations leads to the following differential equation:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} + \frac{D}{I_Z} \cdot \varphi = 0$$

Theory (3/4)

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Using the ansatz $\varphi = \varphi_0 \cos(\omega t)$ one obtains a relation of the swing period $T = 2\pi/\omega$, the angular restoring constant D and the moment of inertia I_Z:

$$T = 2\pi\sqrt{\frac{I_Z}{D}} \quad \Leftrightarrow \quad I_Z = D \cdot \left(\frac{T}{2\pi}\right)^2 \quad \Leftrightarrow \quad D = I_Z \cdot \left(\frac{2\pi}{T}\right)^2 \tag{2}$$

For geometrically shaped rigid bodies the moment of inertia I_Z can also easily be calculated for a given density distribution. If $\rho(x,y,z)$ is the density distribution of the body, the moment of inertia I_Z is obtained by

$$I_Z = \iiint (x^2 y^2)\rho(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

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Theory (4/4) Moments of Inertia

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- A spere of radius r and mass m:

$$I_Z = \frac{2}{5}mr^2$$

- A disc/cylinder of radius r and mass m:

$$I_Z = \frac{1}{2}mr^2$$

- A hollow cylinder of radii r_1 and r_2 and mass m:

$$I_Z = \frac{1}{2}m\left(r_1^2 + r_2^2\right)$$

- A thin rod of length l and mass m:

$$I_Z = \frac{1}{12}ml^2$$

- A point mass m at distance a from the rotation axis:

$$I_Z = ma^2$$

- Steiner's theorem for a rotation axis shifted by a with respect to the known moment of inertia axis:

$$I_Z = I_{Z'} + ma^2$$

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Equipment

Position	Material	Item No.	Quantity
1	measureLAB, multi-user license	14580-61	1
2	Cobra SMARTsense - Rotary Motion (Bluetooth + USB)	12918-01	1
3	Cobra SMARTsense - Force and Acceleration, ± 50N / ± 16g (Bluetooth + USB)	12943-00	1
4	Angular oscillation apparatus	02415-88	1
5	Tripod base PHYWE	02002-55	2
6	Support rod, stainless steel, l = 250 mm, d = 10 mm	02031-00	1
7	Fishing line, l. 20m	02089-00	1
8	Weight holder, silver bronze, 1 g	02407-00	1
9	Slotted weight, silver bronze, 10 g	02205-02	3
10	Measuring tape, l = 2 m	09936-00	1

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Additional equipment

Position	Material	Quantity
1	Portable Balance (e.g. 48921-00)	1
2	Silk thread (e.g. 02412-00)	1 m

Setup and procedure

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Setup (1/7)

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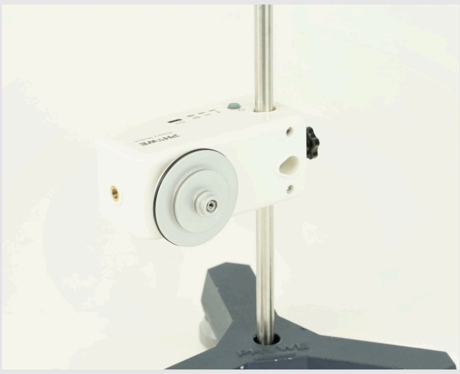
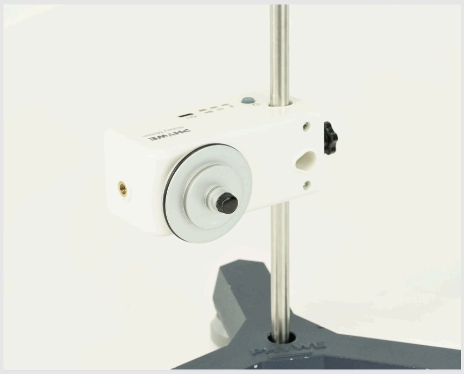
- Mount the Cobra SMARTsense Rotary Motion onto the tripod base with support rod.
- Plug the adapter with different diameters for the belt drive onto the motion sensor pin.
- Fix the adapter with the small knurled screw.





Setup (2/7)

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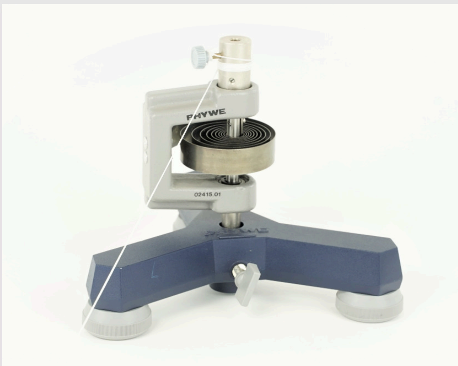
- Mount the rotation axle onto the tripod base. Take a piece of cotton/silk thread (approx. 1 m), knot it on the knurled screw and wind it several times tightly around the rotation axis.
- Now knot the open end of the thread onto the 1g weight holder, wind it around the adapter plate of the rotary motion sensor once, such that the weight holder hangs freely. Add about 20g to the holder.



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Setup (3/7)

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- Mount the bar onto the rotation axle.
- Ensure that the thread that connects the axis of rotation and the wheel of the rotary motion sensor is horizontal and well tightened by the mass on the weight holder. Adjust the height of the sensor or the mass on the holder if necessary.
- The distance between the rotary motion sensor needs to be large enough to guarantee a free swinging of all bodies (especially the bar). In the equillibrium position, the weight holder should hang freely halfway between the table and the wheel of the movement sensor.

Setup (4/7)

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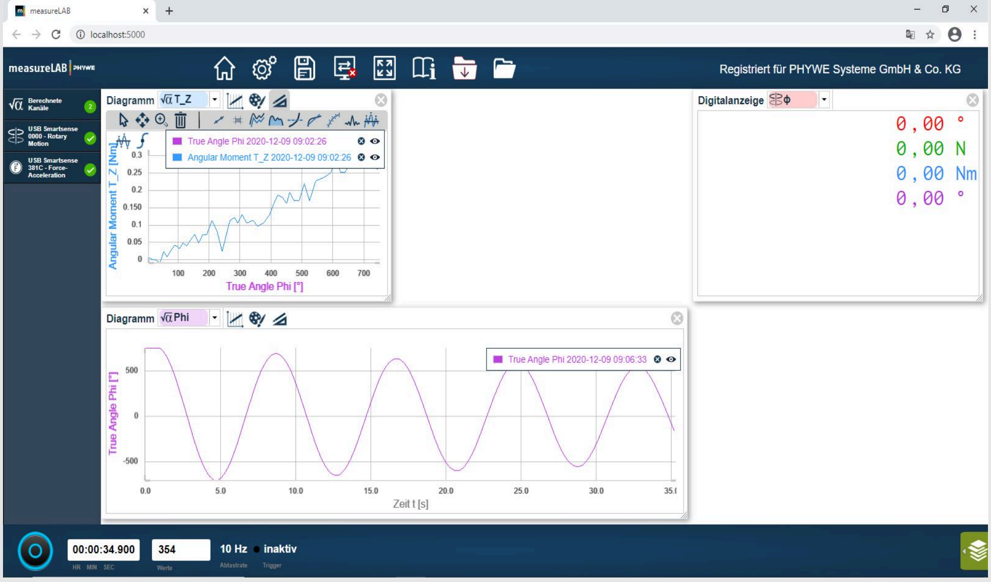
- Turn on the rotary motion as well as the force & acceleration sensor by pressing the on/off button for approximately 3 seconds or simply connect it via USB to the computer.
- Start the measureLAB software. Search for and select the experiment 'P2133167'. The sensor should automatically be activated and your screen should look like this:



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Setup (5/7)

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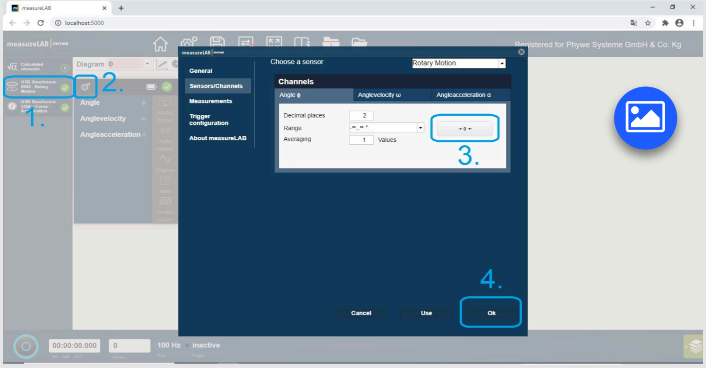
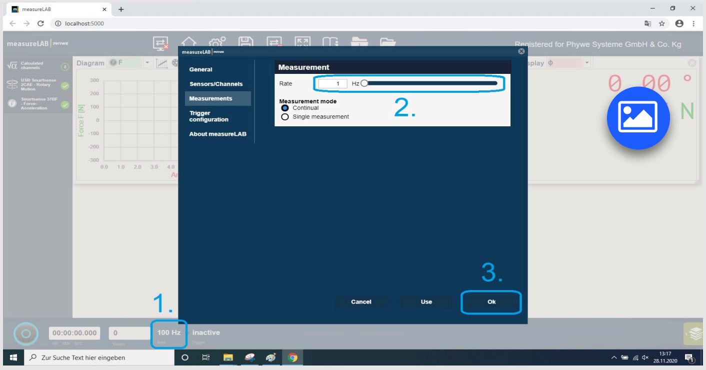
You can also manually set everything for the measurement. Make sure to set both the sensors to zero before every measurement!

The frequency should be set to $1\ Hz$ for the inicial measurement of the spring constant. For the later measurements of the angular oscillations the frequency should be increase to for example $10\ Hz$.





Setup (6/7)

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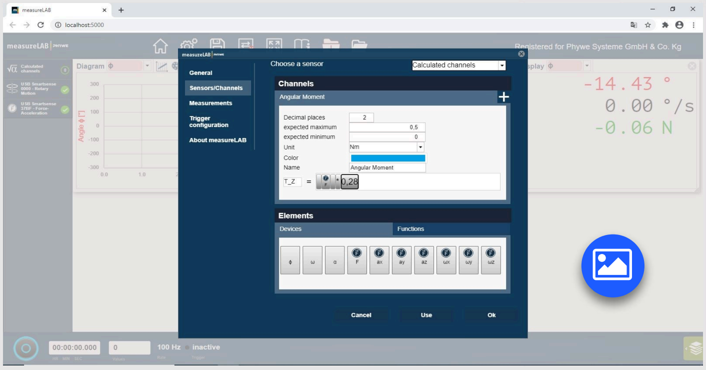
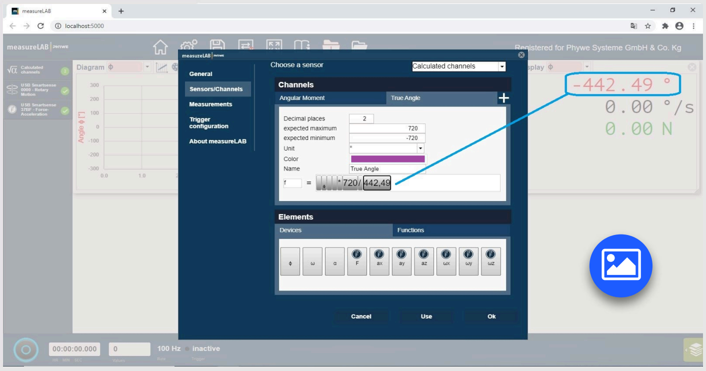
Two virtual channels are set in order to account for the angular moment (instead of the force) and for the true angular deflection which corrects the gear ratio. Set the two masses of the bar at the ends of the bar. The distance (lever) from the axis of rotation to the knurled screw should be about $28\ cm$. For the gear ratio deflect the bar by $720\ ^\circ$ and note the measured angle of the rotary motion sensor.





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Setup (7/7)

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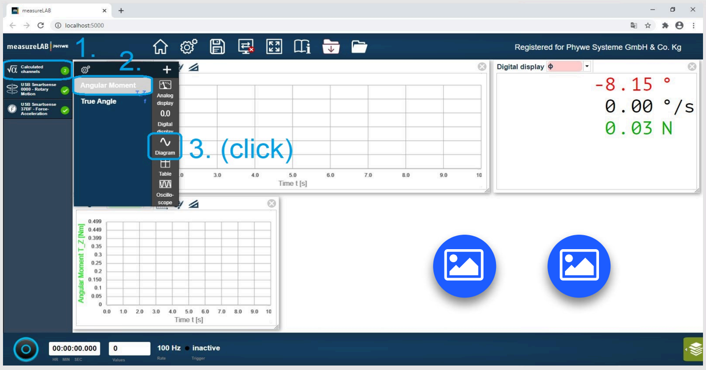
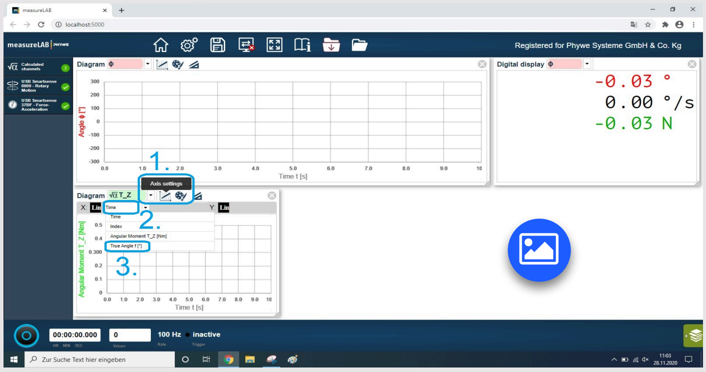
In order to create new diagrams you can at any time select the diagram option from a measurement channel or virtual channel. To add further measurement channels or virtual channels to the diagram simply drag and drop the channel name to the diagram. Instead of the time you can select any channel for the x-axis.





Procedure (1/5)

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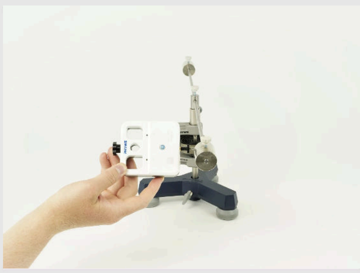

- Adjust the force sensor in horizontal position to zero and hang its hook onto the bar or the knurled screw of one of the masses.

You can either start a measurement and record the dependence $T_Z(\varphi)$ directly or manually determine the acting force for different angles of rotation:

a) Start a measurement at $1\ Hz$. Make sure to always pull exactly perpendicular to the bar. Pull the bar with the force sensor from 0° to 720°. Stop the measurement.

b) Pull the force sensor such that the bar is deflected to 90°, 180°, 270°, 360° and determine the applied force for each position. Make sure that the force sensor is always held at a right angle to the lever arm and in a horizontal position. Note all measured values into table 1 in the protocol.

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Procedure (2/5)

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- Put the bar back to its equilibrium position. Hang the hook next to the left mass and repeat the procedure for angles of -90°, -180°, -270°, -360°. At last measure the length of the lever (hook position - rotation axis).
- Note all measured values into table 1 in the protocol.

Procedure (3/5)

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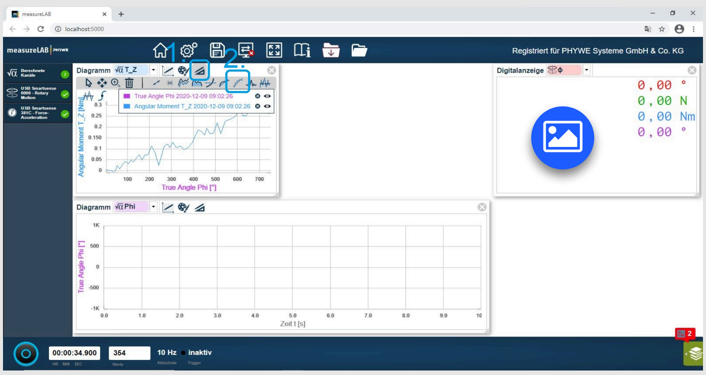
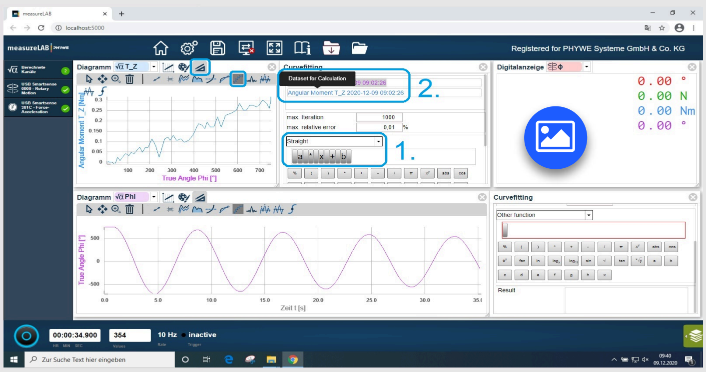
- In case of the continues measurement, use the linear regression tool to determine the slope. Note the resulting value in the table. In this case you don't need to fill the rest of the table.





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Procedure (4/5)

Set the bar to its equilibrium position and mount the two masses each $a = 5\,cm$ (center of mass at knurled screw) apart from the axis of rotation. Deflect the bar by about 360°. Start the measurement and release the bar. Stop the measurement after several periods or for example after 30 seconds. Determine the oscillation period T and note it in the table. Proceed in the same way for distances $a = 10\,cm, 15\,cm, 20\,cm, 25\,cm$

Procedure (5/5)

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- Proceed in the same way for the five rigid bodies. Each period time should be measure 3 times in order to have enough statistics. You can either use the analysis tools to measure the period by hand or use the curve fitting option. Note the resulting values in table 3 of the protocol.

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Evaluation (1/4) Table 1

Note the measured forces F for each deflection angle and the measured length of the lever arm l:

Then calculate the applied torsional moment $T_Z = F \cdot l$ and the angular restoring constant $D = |T_Z/\varphi|$. D is by definition positive. Determine the average value of the angular restoring constant in Nm:

$l =$ ______ cm

$\langle D \rangle =$ ______ Nm

$\varphi\,[^\circ]$	$F\,[N]$	$T_Z\,[Ncm]$	$\varphi\,[rad]$	$D\,[Ncm]$
90				
180				
270				
360				

$\varphi\,[^\circ]$	$F\,[N]$	$T_Z\,[Ncm]$	$\varphi\,[rad]$	$D\,[Ncm]$
-90				
-180				
-270				
-360				

Evaluation (2/4) Table 2

Note the measured periods T_i for each distance a and calculate their average. Then use the averaged periods $\langle T \rangle$ and the inicially determined averaged angular restoring constant $\langle D \rangle$ to calculate the moments of inertia according to equation (2) of the theory. Measure the masses and the dimensions of the bar and calculate the theoretical moments of inertia $I_{Z,th}$ / angular restoring constant D_{th} using the averaged periods again.

$a\,[m]$	$T_1\,[s]$	$T_2\,[s]$	$T_3\,[s]$	$\langle T \rangle\,[s]$	$I_Z\,[kg\,m^2]$	$I_{Z,th}\,[kg\,m^2]$	$D_{th}\,[Nm]$
0.05							
0.10							
0.15							
0.20							
0.25							

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Evaluation (3/4) Table 3

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Note the measured periods T_i for each rigid body and calculate their average. Then use the averaged periods $\langle T \rangle$ and the previously determined averaged angular restoring constant $\langle D \rangle$ to calculate the moments of inertia according to equation (2) of the theory. Measure the masses and dimensions of the bodies and calculate the theoretical moments of inertia $I_{Z,th}$ / angular restoring constant D_{th} using the averaged periods again.

Rigid Body	$T_1\ [s]$	$T_2\ [s]$	$T_3\ [s]$	$\langle T \rangle\ [s]$	$I_Z\ [kg\,m^2]$	$I_{Z,th}\ [kg\,m^2]$	$D_{th}\ [Nm]$
Sphere							
Solid Cylinder							
Hollow Cylinder							
Thick Disc							
Flat Disc							

Evaluation (4/4)

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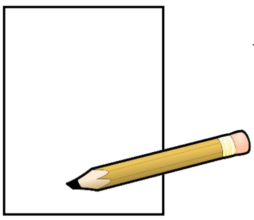
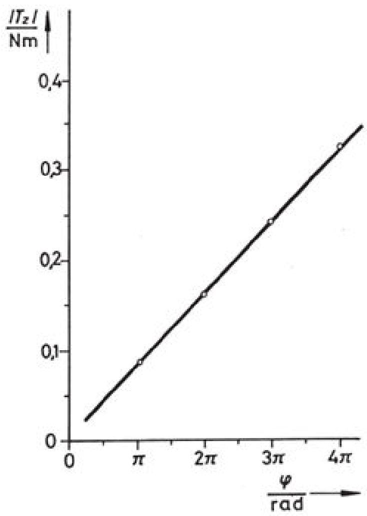
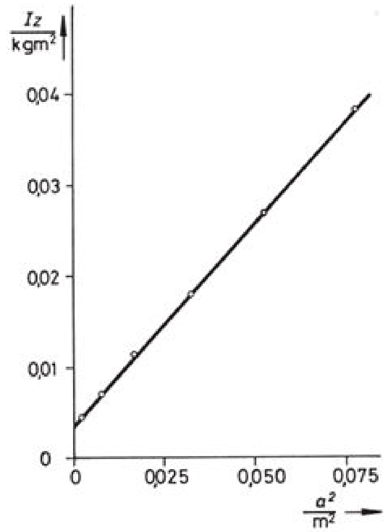
In order to receive an even more accurate result, the measured data should be plotted ($|T_Z|(\varphi)$ and $I_{Z,bar}(a^2)$) and a linear regression should each be applied according to the following equations:

$$|T_Z| = D \cdot \varphi$$

$$I_Z = I_{Z,rod} + (2m) \cdot a^2$$







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