Moments of inertia and torsional vibrations

Physics | Mechanics | Vibrations & waves

Physics | Mechanics | Circular motion & rotation

Applied Science | Engineering | Applied Mechanics | Dynamics

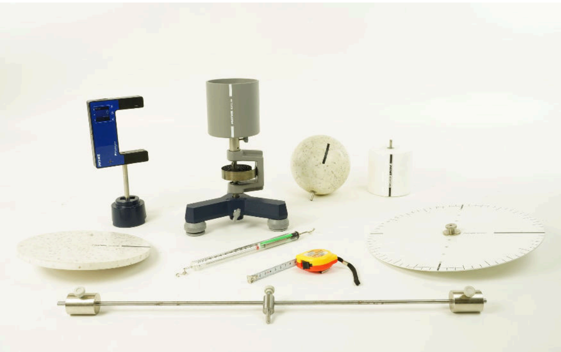
Difficulty level	Group size	Preparation time	Execution time
hard	1	10 minutes	20 minutes

This content can also be found online at:

http://localhost:1337/c/631711bd9ebaee00039a3d75

General information

Application



Experimental setup

The moment of inertia $\hat{I}$ (also commonly $\hat{J}$ or $\hat{\Theta}$) in the case of rotational movements is the counterpart to mass m in linear movements. Together with the angular velocity $\vec{\omega}$, it determines how large the angular momentum $\vec{L}$ or rotational energy E_{rot} of a body is in a rotational movement.

The moment of inertia can be calculated for a body from the volume integral if its density distribution $\rho(\vec{r})$ is known:

$$I = \int_V \vec{r}_\perp^2 \rho(\vec{r}) \mathrm{d}V$$

$\vec{r}_\perp$ corresponds to the component of $\vec{r}$ that is perpendicular to the axis of rotation $\vec{\omega}$.

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Other information (1/2)

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Prior knowledge

Basic knowledge of physical quantities such as momentum, mass and speed as well as the theoretical description of a harmonic oscillation (for example in a mathematical pendulum) should be available. Ideally, terms like angular momentum, moment of inertia and rotational velocity should already be worked out theoretically.

Scientific principle

In the case of rotational movements, different parts of a rigid body move at different velocities. For the mathematical description of a rotational movement, therefore, instead of the mass of the body, its so-called moment of inertia (the inertia tensor) is used as a kind of measure of the density distribution around the axis of rotation.

Other information (2/2)

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Learning objective

After the successful completion of this experiment you will be able to theoretically calculate the moment of inertia of different rigid bodies and also to determine it experimentally from the period of a torsional vibration. You will also gain knowledge about Steiner's theorem.

Tasks

The following will be determined:

1. The angular restoring moment of the spiral spring.
2. The moment of inertia of different ridgid bodies (two discs, two cylinders, a sphere)
3. The moment of inertia of two point masses, as a function of the perpendicular distance to the axis of rotation with its centre of gravity in the axis of rotation.

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Safety instructions

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The general instructions for safe experimentation in science lessons apply to this experiment.

Theory (1/4)

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The relationship between the angular momentum $\vec{L}$ of a rigid body in a stationary coordinate system with its origin at the center of gravity, and the moment $\vec{T}$ acting on it, is given by:

$$\vec{T} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{L}$$

Whereas the angular momentum is expressed by the angular velocity $\vec{\omega}$ and the inertia tensor $\hat{I}$:

$$\vec{L} = \hat{I} \otimes \vec{\omega}$$

For the given rotation around the z-axis of this experiment, the equation for the angular momentum reduces to the z-component L_Z and only depends on the z-component of the moment of inertia I_Z:

$$L_Z = I_Z \cdot \omega$$

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Theory (2/4)

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Insertion into the first equation leads to:

$$T_Z = I_Z \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} = I_Z \cdot \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}$$

with φ being the angle of rotation around the z-axis. On the other hand, according to Hooke's law, the moment T_Z that is necessary to deflect to a spiral spring to a certain angle is defined as

$$T_Z = -D \cdot \varphi \qquad (1)$$

with D being the angular restoring constant of the spring. Comparison of both above equations leads to the following differential equation:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} + \frac{D}{I_Z} \cdot \varphi = 0$$

Theory (3/4)

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Using the ansatz $\varphi = \varphi_0 \cos(\omega t)$ one obtains a relation of the swing period $T = 2\pi/\omega$, the angular restoring constant D and the moment of inertia I_Z:

$$T = 2\pi\sqrt{\frac{I_Z}{D}} \quad \Leftrightarrow \quad I_Z = D \cdot \left(\frac{T}{2\pi}\right)^2 \quad \Leftrightarrow \quad D = I_Z \cdot \left(\frac{2\pi}{T}\right)^2 \qquad (2)$$

For geometrically shaped rigid bodies the moment of inertia I_Z can also easily be calculated for a given density distribution. If $\rho(x, y, z)$ is the density distribution of the body, the moment of inertia I_Z is obtained by

$$I_Z = \iiint (x^2 y^2)\rho(x, y, z)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

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Theory (4/4) Moments of Inertia

- A spere of radius r and mass m:

$$I_Z = \frac{2}{5} m r^2$$

- A disc/cylinder of radius r and mass m:

$$I_Z = \frac{1}{2} m r^2$$

- A hollow cylinder of radii r_1 and r_2 and mass m:

$$I_Z = \frac{1}{2} m \left(r_1^2 + r_2^2 \right)$$

- A thin rod of length l and mass m:

$$I_Z = \frac{1}{12} m l^2$$

- A point mass m at distance a from the rotation axis:

$$I_Z = m a^2$$

- Steiner's theorem for a rotation axis shifted by a with respect to the known moment of inertia axis:

$$I_Z = I_{Z'} + m a^2$$

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Equipment

Position	Material	Item No.	Quantity
1	Angular oscillation apparatus	02415-88	1
2	Tripod base PHYWE	02002-55	1
3	Cobra SMARTsense Dual Photogate - Double light barrier 0 ... ∞ s (Bluetooth + USB)	12945-00	1
4	Barrel base expert	02004-00	1
5	Spring balance,transparent, 2 N	03065-03	1
6	Measuring tape, l = 2 m	09936-00	1

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Additional equipment

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Position	Material	Quantity
1	Portable Balance (e.g. 48921-00)	1
2	Duct tape / Paper	

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Setup and procedure

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Setup (1/2)

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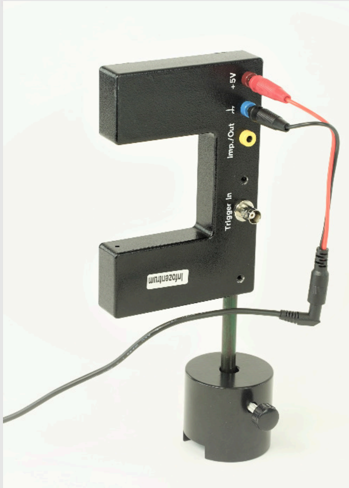
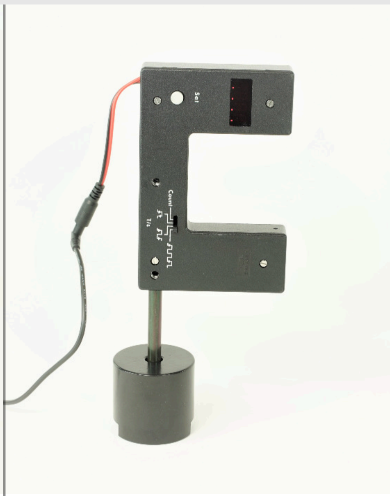


Back view



Front view

- Mount the light barrier onto the barrel base.
- Connect the power supply to the $+5$ and ground sockets of the light barrier.
- Select the fourth position of the switch in order to measure the full period.
- Press the set button to activate the light barrier for each single measurement.

Setup (2/2)

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- Mount the rotation axle onto the tripod base.
- Mount a piece of duct tape (approx. 3 mm wide) onto the five rigid bodies, such that it is long enough to go through the light barrier while swinging.
- For the bar with masses, no additional duct tape is needed.
- Mount the bar with masses onto the rotation axle and fix the masses at both ends.

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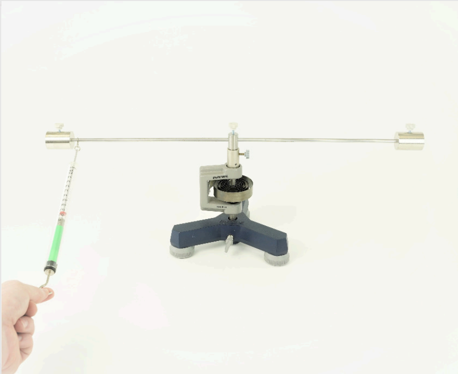
Procedure (1/4)

- Adjust the springbalance in horizontal position to zero.
- Hang its hook onto the bar right next to the right mass.
- Pull the spring balance such that the bar is deflected to 90°, 180°, 270°, 360° and read the scale of the spring balance for each position.
- Make sure that the spring balance is always held at a right angle to the lever arm and in a horizontal position.
- Note all measured values into table 1 in the protocol.

Procedure (2/4)

- Put the bar back to its equilibrium position. Hang the hook next to the left mass and repeat the procedure for angles of -90°, -180°, -270°, -360°. At last measure the length of the lever (hook position - rotation axis).
- Note all measured values into table 1 in the protocol.



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Procedure (3/4)

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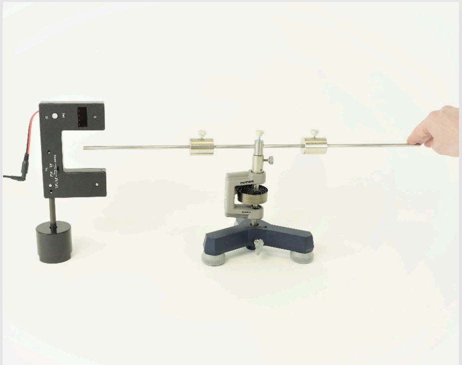
- Mount one of the five rigid bodies to the rotation axle and set it to its equilibrium position.
- Push the light barrier over the duct tape such that the light barrier marks the equilibrium position. Do not move the light barrier anymore.
- Deflect the rigid body by about 360° and release it.
- Press the 'Set' button on the light barrier. Note the resulting period in table 2 of the protocol.
- While the body swings, reset the light barrier at least twice again and note the resulting values for the period.
- Proceed in the same way for the other four rigid bodies.

Procedure (4/4)

Mount the bar with two masses to the rotation axle and set it to its equilibrium position. Set the two masses each $a = 5\,cm$ apart from the axis of rotation. Now measure the period of oscillation T as before. Proceed in the same way for distances $a = 10\,cm, 15\,cm, 20\,cm, 25\,cm$

Note the resulting values in table 3 of the protocol.



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Evaluation (1/4) Table 1

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Note the measured forces F for each deflection angle and the measured length of the lever arm l:

Then calculate the applied torsional moment $T_Z = F \cdot l$ and the angular restoring constant $D = |T_Z/\varphi|$. D is by definition positive. Determine the average value of the angular restoring constant in Nm:

$l =$ ______ cm

$\langle D \rangle =$ ______ Nm

$\varphi\,[^\circ]$	$F\,[N]$	$T_Z\,[Ncm]$	$\varphi\,[rad]$	$D\,[Ncm]$
90				
180				
270				
360				

$\varphi\,[^\circ]$	$F\,[N]$	$T_Z\,[Ncm]$	$\varphi\,[rad]$	$D\,[Ncm]$
-90				
-180				
-270				
-360				

Evaluation (2/4) Table 2

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Note the measured periods T_i for each rigid body and calculate their average. Then use the averaged periods $\langle T \rangle$ and the previously determined averaged angular restoring constant $\langle D \rangle$ to calculate the moments of inertia according to equation (2) of the theory. Measure the masses and dimensions of the bodies and calculate the theoretical moments of inertia $I_{Z,th}$ / angular restoring constant D_{th} using the averaged periods again.

Rigid Body	$T_1\,[s]$	$T_2\,[s]$	$T_3\,[s]$	$\langle T \rangle\,[s]$	$I_Z\,[kg\,m^2]$	$I_{Z,th}\,[kg\,m^2]$	$D_{th}\,[Nm]$
Sphere							
Solid Cylinder							
Hollow Cylinder							
Thick Disc							
Flat Disc							

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Evaluation (3/4) Table 3

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Note the measured periods T_i for each distance a and calculate their average. Then use the averaged periods $\langle T \rangle$ and the inicially determined averaged angular restoring constant $\langle D \rangle$ to calculate the moments of inertia according to equation (2) of the theory. Measure the masses and the dimensions of the bar and calculate the theoretical moments of inertia $I_{Z,th}$ / angular restoring constant D_{th} using the averaged periods again.

$a\,[m]$	$T_1\,[s]$	$T_2\,[s]$	$T_3\,[s]$	$\langle T \rangle\,[s]$	$I_Z\,[kg\,m^2]$	$I_{Z,th}\,[kg\,m^2]$	$D_{th}\,[Nm]$
0.05							
0.10							
0.15							
0.20							
0.25							

Evaluation (4/4)

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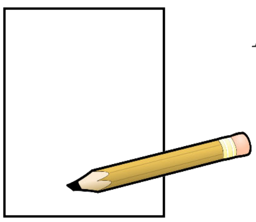
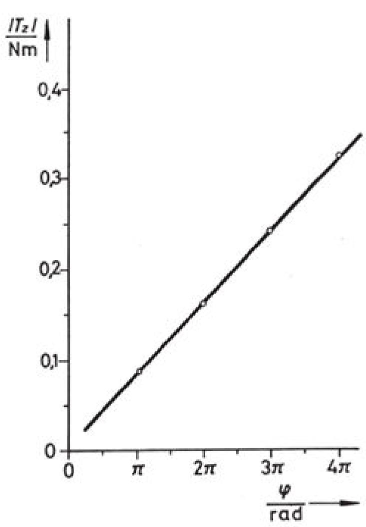
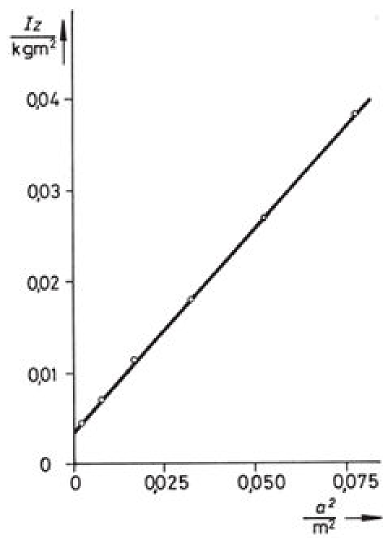
In order to receive an even more accurate result, the measured data should be plotted ($|T_Z|(\varphi)$ and $I_{Z,bar}(a^2)$) and a linear regression should each be applied according to the following equations:

$$|T_Z| = D \cdot \varphi$$



$$I_Z = I_{Z,rod} + (2m) \cdot a^2$$





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