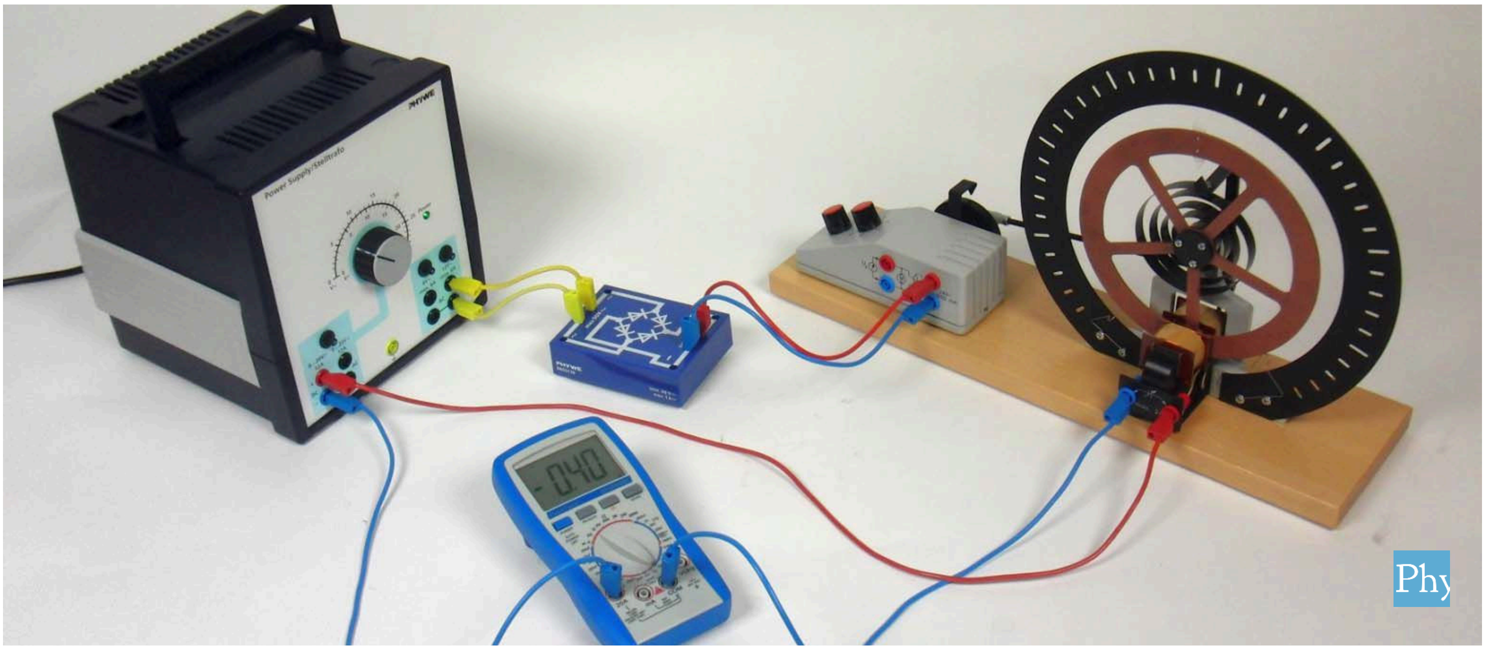
# Forced oscillations - Pohl's pendulum



The goal of this experiment is to investigate the oscillation behaviour induced by forced oscillation.

Physics | Acoustics | Wave Motion

| Difficulty level | Group size | Preparation time | Execution time |
| --- | --- | --- | --- |
| medium | - | - | - |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# General information

## Application

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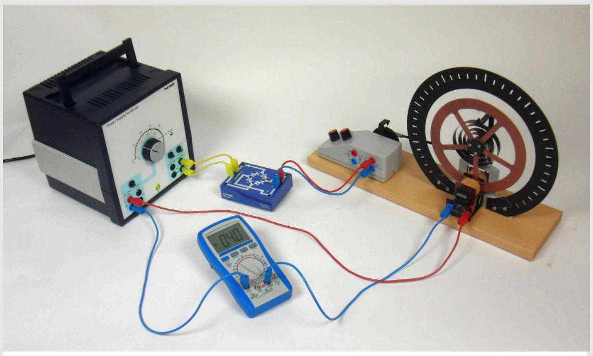

Experimental setup

Pendulum oscillations offer a first understanding of mechanical systems close to the harmonic oscillator, which is fundamental in the description of many physical systems in fields such as particle physics and solid state physics.

This experiment investigates the behaviour of a system, that is forced into oscillation. It offers insight in phenomena such as resonance.



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## Other information (1/3)

### Prior knowledge

The behaviour of a singular pendulum shoud be known.

### Main principle

If an oscillating system is allowed to swing freely it is observed that the decrease of successive maximum amplitudes is highly dependent on the damping. If the oscillating system is stimulated to swing by an external periodic torque, we observe that in the steady state the amplitude is a function of the frequency and the amplitude of the external periodic torque and of the damping. The characteristic frequencies of the free oscillation as well as the resonance curves of the forced oscillation for different damping values are to be determined.

## Other information (2/3)

### Learning objective

The goal of this experiment is to investigate the oscillation behaviour induced by forced oscillation.

### Tasks

**Free oscillation**

1. Determine the oscillating period and the characteristic frequency of the undamped case.
2. Determine the oscillating periods and the corresponding characteristic frequencies for different damping values. The corresponding ratios of attenuation, the damping constants and the logarithmic decrements are to be calculated.
3. Realize the aperiodic and the creeping case.



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## Other information (3/3)

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### Tasks

### Forced oscillation

1. Determine the resonance curves and represent them graphically using the damping values of A. Determine the corresponding resonance frequencies and compare them with the resonance frequency values found beforehand.
2. Observe the phase shifting between the torsion pendulum and the stimulating external torque for a small damping value for different stimulating frequencies

## Theory (1/3)

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### Undamped and damped free oscillation

In case of free and damped torsional vibration torques $M_1$ (spiral spring) and $M_2$ (eddy current brake) act on the pendulum. We have

$M_1 = -D^0 \Phi$ and $M_2 = -C\dot{\Phi}$

$\Phi$ = angle of rotation. $\dot{\Phi}$ = angular velocity, $D^0$ = torque per unit angle, C = factor of proportionality depending on the current which supplies the eddy current brake.

The resultant torque $M_1 = -D^0 \Phi - C\dot{\Phi}$

leads us to the following equation of motion:

$I\ddot{\Phi} + c\dot{\Phi} + D^0 \Phi = 0$ (1)



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# Theory (2/3)

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Dividing Eq. (1) by and using the abbreviations

$\delta = \frac{C}{2I}$ and $\omega_0^2 = \frac{D^0}{I}$

results in $\ddot{\Phi} + 2\delta\Phi + \omega_0^2\Phi = 0$ (2)

$\delta$ is called the “damping constant” and

$\omega_0 = \sqrt{\frac{D^0}{I}}$

$F = m\omega^2 r$

the characteristic frequency of the undamped system.

The solution of the differential equation (2) is

$\Phi(t) = \Phi_0 e^{-\delta t} \cos \omega t$ (3)

$\omega = \sqrt{\omega_0^2 - \delta^2}$ (4)

# Theory (3/3)

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**Forced oscillation**

If the pendulum is acted on by a periodic torque $M_a = M_0 \cos \omega_a t$ Eq. (2) changes into

$\ddot{\Phi} + 2\delta\dot{\Phi} + \omega_0^2\Phi = F_0 \cos \omega_a t$ (7)

where $F_0 = \frac{M_0}{I}$

In the steady state, the solution of this differential equation is

$\Phi(t) = \Phi_a \cos(\omega_a t - \alpha$ (8)

where

$\Phi_\alpha = \frac{\Phi_0}{\sqrt{\left(1 - \left(\frac{\omega_\alpha}{\omega_0}\right)^2\right)^2 + \left(2\frac{\delta}{\omega_0}\frac{\omega_\alpha}{\omega_0}\right)^2}}$ (9)

and $\Phi_\alpha = \frac{F_0}{\omega_0^2}$

Furthermore:

$\tan \alpha = \frac{2\delta\omega_\alpha}{\omega_0^2 - \omega_a^2}$ (10)



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# Equipment

| Position | Material | Item No. | Quantity |
|---|---|---|---|
| 1 | Torsion pendulum after Pohl | 11214-00 | 1 |
| 2 | Digital Laboratory Power Supply 2 x 0 - 30 V/0 - 5 A DC/5 V/3 A fixed | EAK-P-6145 | 1 |
| 3 | Digital stopwatch, 24 h, 1/100 s and 1 s | 24025-00 | 1 |
| 4 | Connecting cord, 32 A, 750 mm, red | 07362-01 | 2 |
| 5 | Connecting cord, 32 A, 750 mm, blue | 07362-04 | 2 |



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# Setup and Procedure

## Setup

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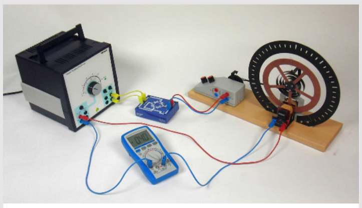

Fig. 1: Experimental setup

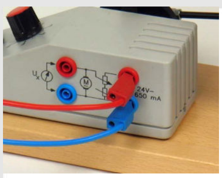

Fig. 3:

Connection of the DC motor and power supply.

The experiment is set up as shown in Fig. 1 und 2. The DC output of the power supply unit is connected to the eddy current brake. The motor also needs DC voltage. For this reason a rectifier is inserted between the AC output (12 V) of the power supply unit and to the two right sockets of the DC motor (see Fig. 3). The DC current supplied to the eddy current brake, $I_B$, is set with the adjusting knob of the power supply and is indicated by the ammeter.

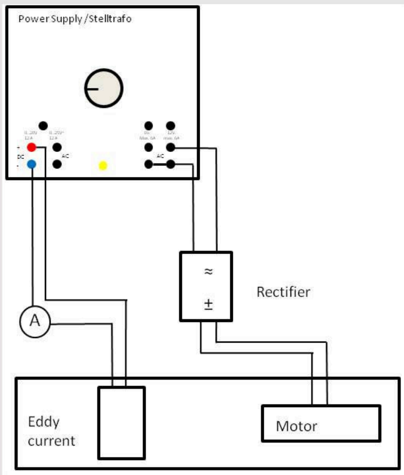


Fig. 2: Electrical connection of the experiment.



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## Procedure (1/5)

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**A. Free oscillation**

**1. Determine the oscillating period and the characteristic frequency of the undamped case**

To determine the characteristic frequency $\omega_0$ of the torsion pendulum without damping ($I_B$ = 0),

- deflect the pendulum completely to one side,
- measure the time for several oscillations.

The measurement is to be repeated several times and the mean value of the characteristic period $\bar{T}_0$ is to be calculated.

## Procedure (2/5)

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**2. Determine the oscillating periods and the corresponding characteristic frequencies for different damping values.**

In the same way the characteristic frequencies for the damped oscillations are found using the following current intensities for the eddy current brake:

$I_B \sim 0.25\,\mathrm{A},\ (\mathrm{U} = 4\,\mathrm{V})$, $I_B \sim 0.40\,\mathrm{A},\ (\mathrm{U} = 6\,\mathrm{V})$, $I_B \sim 0.55\,\mathrm{A},\ (\mathrm{U} = 8\,\mathrm{V})$, $I_B \sim 0.90\,\mathrm{A},\ (\mathrm{U} = 12\,\mathrm{V})$

To determine the damping values for the above mentioned cases measure unidirectional maximum amplitudes as follows:

- deflect the pendulum completely to one side,
- observe the magnitude of successive amplitudes on the other side.



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## Procedure (3/5)

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Initially it has to be ensured that the pendulum pointer at rest coincides with the zero-position of the scale (see Fig. 4). This can be achieved by turning the eccentric disc of the motor.

**3. Realize the aperiodic and the creeping case**

To realize the aperiodic case ($I_B \sim$ 2.0 A) and the creeping case ($I_B \sim$ 2.3 A) the eddy current brake is briefly loaded with more than 2.0 A. **Caution**: Do not use current intensities above 2.0 A for the eddy current brake for more than a few minutes.


Fig. 4: Pendulum pointer at zero-position.

## Procedure (4/5)

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**B. Forced oscillation**

To stimulate the torsion pendulum, the connecting rod of the motor is fixed to the upper third of the stimulating source. The stimulating frequency $\omega_\alpha$ of the motor can be found by using a stopwatch and counting the number of turns (for example: stop the time of 10 turns).

**1. Determine the resonance curves and represent them graphically using the damping values of A.**

The measurement begins with small stimulating frequencies $\omega_\alpha$. $\omega_\alpha$ is increased by means of the motorpotentiometer setting “coarse“. In the vicinity of the maximum $\omega_\alpha$ is changed in small steps using the potentiometer setting “fine“ (see Fig. 5). In each case, readings should only be taken after a stable pendulum amplitude has been established. In the absence of damping or for only very small damping values, $\omega_\alpha$ must be chosen in such a way that the pendulum does not exceed its scale range.



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## Procedure (5/5)

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Fig. 5: Control knobs to set the motor potentiometer. Upper knob: “coarse”; lower knob: “fine”.

**2. Observe the phase shifting between the torsion pendulum and the stimulating external torque for a small damping value for different stimulating frequencies**

Chose a small damping value and stimulate the pendulum in one case with a frequency $\omega_\alpha$ far below the resonance frequency and in the other case far above it. Observe the corresponding phase shifts between the torsion pendulum and the external torque. In each case, readings should only be taken after a stable pendulum amplitude has been established.

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# Evaluation



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## Evaluation (1/6)

The mean value of the period $\bar{t}_0$ and the corresponding characteristic frequency $\omega_0$ of the free and undamped swinging torsional pendulum are found to be

$\bar{T}_0 = (1.817 \pm 0.017)$s; $\frac{\Delta \bar{T}_0}{\bar{T}_0} = \pm 1\%$

and $\bar{\omega}_0 = (3.46 \pm 0.03)$1/s

## Evaluation (2/6)

Plot successive unidirectional maximum amplitudes as a function of time. The respective time is calculated from the frequency. See Fig 6 for sample results. The corresponding ratios of attenuation, the damping constants K and the logarithmic decrements $\Lambda$ are to be calculated as follows:
Eq. (3) shows that the amplitude $\Phi(t)$ of the damped oscillation has decreased to the e-th part of the initial amplitude $\Phi_0$ after the time $t = 1/\delta$ has elapsed. Moreover, from Eq. (3) it follows that the ratio of two successive amplitudes is constant.

$\frac{\Phi_n}{\Phi_{n+1}} = K = e^{\delta T}$ (5)

K is called the “damping ratio” and T = oscillating period and the quantity

$\Lambda = \ln K = \delta T = \ln \frac{\Phi_n}{\Phi_{n+1}}$ (6)

is called the “logarithmic decrement“.



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## Evaluation (3/6)

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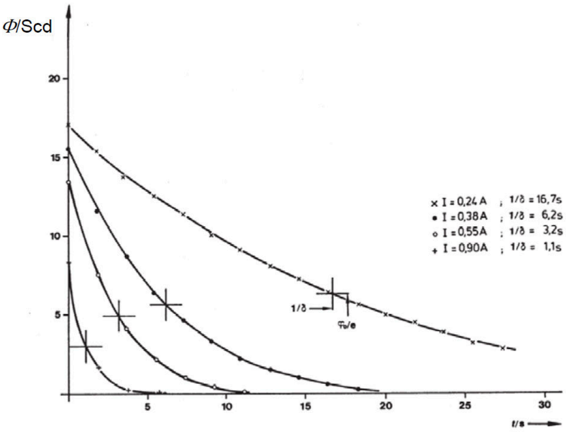


Fig. 6: Maximum values of unidirectional amplitudes as a function of time for different dampings.

Sample results for the characteristic damping values:

| **I [A]** | **1/$\delta$ [s]** | $\delta$ **[1/s]** | $\omega = \sqrt{\omega_0^2 - \delta^2}$ **[1/s]** | $K = \frac{\Phi_n}{\Phi_{n+1}}$ | $\Lambda$ |
|---|---|---|---|---|---|
| 0.25 | 16.7 | 0.06 | 3.46 | 1.1 | 0.12 |
| 0.4 | 6.2 | 0.16 | 3.45 | 1.4 | 0.31 |
| 0.55 | 3.2 | 0.31 | 3.44 | 1.9 | 0.64 |
| 0.9 | 1.1 | 0.91 | 3.34 | 5.6 | 1.72 |

## Evaluation (4/6)

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Eq. (4) has a real solution only if $\omega_0^2 \geq \delta^2$. For $\omega_0^2 = \delta^2$, the pendulum returns in a minimum of time to its initial position without oscillating (aperiodic case). For $\omega_0^2 \leq \delta^2$ the pendulum returns asymptotically to its initial position (creeping).



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## Evaluation (5/6)

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Fig. 7 shows the resonance curves for different dampings. An analysis of Eq. (9) gives evidence of the following which is confirmed by the results in Fig 7:

1. The greater $F_0$, the greater $\Phi_\alpha$
2. For a fixed value $F_0$ we have: $\Phi \rightarrow \Phi_{max}$ for $\omega_\alpha = \omega_0$
3. The greater $\delta$, the smaller $\Phi_\alpha$
4. For $\delta = 0$ we find $\Phi_a \rightarrow \infty$ if $\omega_\alpha = \omega_0$

Evaluating the curves of Fig 7 leads in this sample result to medium resonance frequency of $\omega = 3.41$ 1/s which comes very close to the resonance frequency determined in task A1.

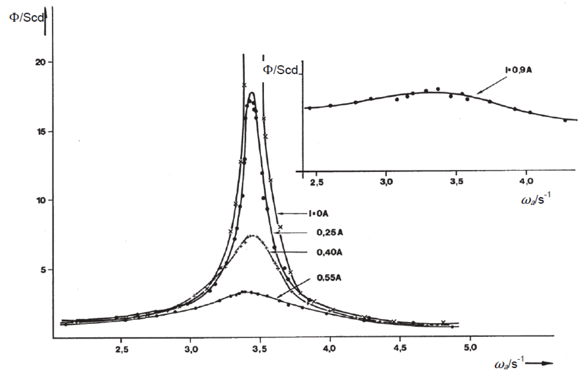


Fig. 7: Resonance curves for different dampings.

## Evaluation (6/6)

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Fig. 8 shows the phase difference of the forced oscillation as a function of the stimulating frequency according to Eq. (10). For very small frequencies $\omega_\alpha$ the phase difference is approximately zero, i.e. the pendulum and the stimulating torque are “inphase”.

If $\omega_\alpha$ is much greater than $\omega_0$, pendulum and stimulating torque are nearly in opposite phase to each other. The smaller the damping, the faster the transition from swinging “inphase” to the “in opposite phase” state can be achieved.

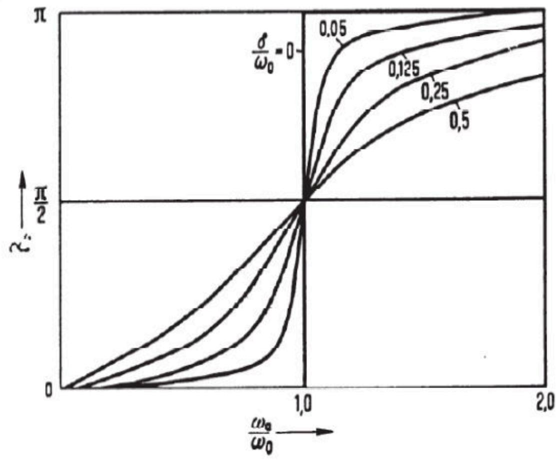


Fig. 8: Phase shifting of forced oscillation for different dampings.



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