Moment of inertia and angular acceleration with a precision pivot bearing

Physics | Mechanics | Circular motion & rotation

Applied Science | Engineering | Applied Mechanics | Dynamics

Difficulty level	Group size	Preparation time	Execution time
hard	2	45+ minutes	45+ minutes

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General information

Application

Fig 1: Experimental set-up with turntable

The moment of inertia is used widely in mechanics and as such had wide applications in engineering but it is also used prominetly in fundamental research in all fields of physics.

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Other information (1/2)

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Prior knowledge

The prior knowledge for this experiment is found in the Theory section.

Main principle

A known torque is applied to a body that can rotate about a fixed axis with minimal friction. Angle and angular velocity are measured over the time and the moment of inertia is determined. The torque is exerted by a string on a wheel of known radius with the force on the string resulting from the known force of a mass in the earth's gravitational field. The known energy gain of the lowering mass is converted to rotational energy of the body under observation.

Other information (2/2)

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Learning objective

The goal of this experiment is to investigate moment of inertia and angular acceleration.

Tasks

1. Measure angular velocity and angle of rotation vs. time for a disc with constant torque applied to it for different values of torque generated with various forces on three different radii. Calculate the moment of inertia of the disc.
2. Measure angular velocity and angle of rotation vs. time and thus the moment of inertia for two discs and for a bar with masses mounted to it at different distances from the axis of rotation.
3. Calculate the rotational energy and the angular momentum of the disc over the time. Calculate the energy loss of the weight from the height loss over the time and compare these two energies.

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Theory (1/6)

The angular momentum $\vec{J}$ of a single particle at place $\vec{r}$ with velocity $\vec{v}$,mass m and momentum $\vec{p} = m\vec{v}$ is defined as

$\vec{J} = \vec{r} \times \vec{p}$

and the torque $\vec{T}$ from the force $\vec{F}$ is defined as

$\vec{T} = \vec{r} \times \vec{F}$

with torque and angular momentum depending on the origin of the reference frame. The change of $\vec{J}$ in time is

$\frac{\mathrm{d}\vec{J}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(\vec{r} \times \vec{p}) = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \times \vec{p} + \vec{r} \times \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$

Theory (2/6)

and with $\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \times \vec{p} = \vec{v} \times m\vec{v} \equiv 0$

and Newton's law $\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$
the equation of motion becomes $\vec{T} = \frac{\mathrm{d}\vec{J}}{\mathrm{d}t}$ (1)

For a system of N particles with center of mass $\vec{R}_{\text{c.m.}}$ and total linear momentum $\vec{P} = \sum m_i \vec{v}_i$

$\vec{J} = \sum_{i=1}^{N} m_i \left(\vec{r}_i - \vec{R}_{\text{c.m.}}\right) \times \vec{v}_i + \sum_{i=1}^{N} m_i \vec{R}_{\text{c.m.}} \times \vec{v}_i = \vec{J}_{\text{c.m.}} + \vec{R}_{\text{c.m.}} \times \vec{P}$

Now the movement of the center of mass is neglected, the origin set to the center of mass and a rigid body assumed with $\vec{r}_i - \vec{r}_j$ fixed. The velocity of particle i may be written as $\vec{v}_i = \vec{\omega} \times \vec{r}_i$ with vector of rotation

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Theory (3/6)

$\vec{\omega} = \frac{\mathrm{d}\vec{\varphi}}{\mathrm{d}t}$ (2) constant throughout the body. Then

$\vec{J} = \sum m_i \vec{r}_i \times \vec{v}_i = \sum m_i \vec{r}_i \times (\vec{\omega} \times \vec{r}_i)$

With $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{b}\,(\vec{a} \cdot \vec{c}) - \vec{c}\left(\vec{a} \cdot \vec{b}\right)$.

$\vec{J} = \sum m_i \left(\vec{\omega} \cdot r_i^2 - \vec{r}_i\,(\vec{r}_i \cdot \vec{\omega})\right)$

with $\vec{r}_i \cdot \vec{\omega} = x_i + \omega_x + y_i + \omega_y + z_i + \omega_z$ and
$J_z = \omega_z \sum m_i \left(r_i^2 - z_i^2\right) - \omega_z \sum m_i z_i x_i - \omega_y \sum m_i z_i y_i$

The inertial coefficients or moments of inertia are defined as

$I_{x,x} = \sum m_i \left(r_i^2 - x_i^2\right)$ (3)

$I_{x,y} = -\sum m_i x_i y_i$

$I_{x,z} = -\sum m_i x_i z_i$

Theory (4/6)

and with the matrix $\hat{I} = \{I_{k,l}\}$ it is $\vec{J} = \hat{I} \cdot \vec{\omega}$ (4)

and for the rotational acceleration $\vec{\alpha} = \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t}$ it is

$\vec{T} = \frac{\mathrm{d}\vec{J}}{\mathrm{d}t} = \hat{I} \cdot \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \hat{I} \cdot \vec{\alpha}.$

The rotational energy is

$E = \frac{1}{2} \sum m_i v_i^2 = \frac{1}{2} \sum m_i \left(\vec{\omega} \times \vec{r_i}\right)^2 = \frac{1}{2} I_{k,l} \omega_k \omega_l$

Sum convention: sum up over same indices using

$\left(\vec{a} \times \vec{b}\right)\left(\vec{c} \times \vec{d}\right) = (\vec{a} \cdot \vec{c})\left(\vec{b} \cdot \vec{d}\right) - \left(\vec{a} \cdot \vec{d}\right)\left(\vec{b} \cdot \vec{c}\right)$

The coordinate axes can always be set to the "principal axes of inertia" so that none but the diagonal elements of the matrix $I_{k,k} \neq 0$. In this experiment only a rotation about the z-axis can occur and $\vec{\omega} = \hat{e}_z \omega_z = \hat{e}_z \omega$ with the unit vector $\hat{e}_z$. The energy is then

$E = \frac{1}{2} I_{z,z} \omega^2$

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Theory (5/6)

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The torque $T = m_a(g-a)r_a$ is nearly constant in time since the acceleration $a = \alpha \cdot r_a$ of the mass m_a used for accelerating the rotation is small compared to the gravitational acceleration g = 9.81 m/s² and the thread is always tangential to the wheel with r_a. So with (1), (2) and (3)

$$T = I_{z,z} \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = m_a g r_a \qquad (6)$$

$$\omega(t) = \omega(t=0) + \frac{m_a g r_a}{I_{z,z}} \cdot t = \omega(t=0) + \frac{T}{I_{z,z}} \cdot t \qquad (7)$$

$$\varphi(t) = \varphi(t=0) + \frac{1}{2} \frac{m_a g r_a}{I_{z,z}} \cdot t^2 = \varphi(t=0) + \frac{1}{2} \frac{T}{I_{z,z}} \cdot t^2 \qquad (8)$$

Theory (6/6)

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The potential energy of the accelerating weight is

$$E = m_a g h(t) = -m_a g(\varphi(t) \cdot r_a \overset{(8)}{=} \frac{1}{2} \frac{m_a^2 g^2 r_a^2}{I_{z,z}} \cdot t^2 \overset{(7)}{=} \frac{1}{2} I_{z,z}\omega^2$$

thus verifying (4).

If a weight m_i is mounted to a rod in a distance r_i to the fixed axis z, which is the axis of rotation perpendicular to the rod, with the rod lying along the y-axis, then the coordinates of the weight are (0, r_i, 0). According to (3), the moment of inertia about the z-axis is $I_{z,z} = m_i r_i^2$, about the x-axis $I_{x,x} = m_i r_i^2$ and about the y-axis it is $I_{y,y} = 0$.

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Equipment

Position	Material	Item No.	Quantity
1	Tripod base PHYWE	02002-55	1
2	Precision pivot bearing	02419-00	1
3	Inertia rod	02417-03	1
4	Turntable with angle scale	02417-02	1
5	Aperture plate for turntable	02417-05	1
6	Connecting cord, 32 A, 1000 mm, red	07363-01	1
7	Connecting cord, 32 A, 1000 mm, blue	07363-04	1
8	Light barrier with counter	11207-30	1
9	Adapter, BNC male/4 mm female pair	07542-26	1
10	Capacitor 100 nF/250V, G1	39105-18	1
11	Power supply 5 V DC/2.4 A with 4 mm plugs	11077-99	1
12	Precision pulley	11201-02	1
13	Bench clamp expert	02011-00	2
14	Silk thread, l = 200 m	02412-00	1
15	Circular level, d = 36 mm	02123-00	1
16	Weight holder, 10 g	02204-00	1
17	Slotted weight, black, 10 g	02205-01	10
18	Slotted weight, black, 50 g	02206-01	2
19	Weight holder, silver bronze, 1 g	02407-00	1
20	Slotted weight, blank, 1 g	03916-00	20
21	Holding device with cable releasee	02417-04	1
22	Measuring tape, l = 2 m	09936-00	1
23	Barrel base expert	02004-00	1

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Setup and Procedure

Setup (1/3)

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Set the experiment up as seen in Fig. 1. Connect the light barrier with counter according to Fig. 2 to the cable release. Adjust the turntable to be horizontal – it must not start to move with an imbalance without other torque applied. Fix the silk thread (with the weight holder on one end) with the screw of the precision bearing or a piece of adhesive tape to the wheels with the grooves on the axis of rotation. Wind it several times around one of the wheels – enough turns, that the weight may reach the floor. Be sure the thread and the wheel of the precision pulley and the groove of the selected wheel are well aligned. Place the holding device with cable release in a way that it just holds the turntable on the sector mask and does not disturb the movement after release.

Note down the angle ϕ rom the point of release to the point where the aperture plate enters the beam of the light barrier and the radius r_a of the selected wheel with groove.

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Setup (2/3)

Measure the time t_1 necessary for the turntable to rotate about the angle ϕ by setting the mode selection switch of the light barrier to the symbol . Press the reset button on the light barrier. Release the holder – the clock should start counting. To enable the counter to stop on the interruption of the light beam, the cable release has to be pushed back into it's holding position. After pushing the cable release back the counter should stop, when the mask enters the light beam.

Measure the time necessary for the angle $\phi + 2\pi$ by pushing the release back only after the mask has completed a whole turn and has passed again and so on for two and more turns. (Catch the mask by hand before it hits the holder.)

Setup (3/3)

Measure the angular velocity ω after a rotation about the angle ϕ by measuring the time t_2 that the mask needs to pass the light barrier: Set the mode selection switch to the symbol and after releasing the holder the counter starts counting. Press the reset button before the mask enters the light barrier. Then the counter starts when the mask enters the light beam and counts as long as the light beam is interrupted. If the measured time was t_2 and the sector mask covers the angle 15°, then the radian of this angle is $\varphi = 2 \cdot \pi \cdot 15/360$ and the frequency or angular velocity $\omega = \varphi / t_2$. You may press the reset button after one or more passes of the mask measuring the angular velocity after more than one turn of the rotation.

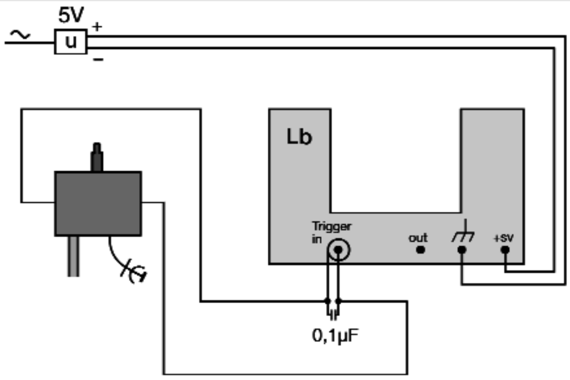

Fig 2: Connection of the light barrier (Lb).

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Procedure (1/2)

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Take measurements with different accelerating weights ma up to 100 g.

Note down measurement values with the thread running in different grooves, i. e. different radii r_a for the accelerating torque – adjust the position of the precision pulley to align thread and groove and wheel of the pulley (thread has to be horizontal). Choose especially the weight on the end of the thread m_a such that the torque $\vec{T} = \vec{r}_a \times \vec{F}$ is constant – e. g. m_a = 60 g for r_a = 15 mm and m_a = 30 g for r_a = 30 mm and m_a = 20 g for r_a = 45 mm, each time the torque being $T = m_a g r$ = 8.83 mNm with earth's gravitational acceleration g = 9.81 N/kg=9.81 m/s². Also choose a weight with which you take a measurement for each groove radius.

You may also take a measurement with no weight m_a.

Start the turntable by a shove with the hand. Nearly unaccelerated movement should be observable. Also measure the angular velocity after converting a defined amount of potential energy to rotational energy by allowing the weight m_a to pass only a given height before touching the ground.

Procedure (2/2)

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Record measurements with two turntables mounted on the precision bearing with weight values used on the single turntable and with double the weight used on the single turntable for comparison.

Remove the turntables and mount the inertia rod to the precision bearing and the two weight holders symmetrically to the rod with both the same distance to the axis r_i.

Take measurements with various masses m_i at constant r_i and also with constant masses m_i at varied r_i (both masses of course still mounted symmetrically) – accelerated with the same weight m_a(or with the same series of weights for high precision).

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Evaluation

Evaluation (1/5)

For evaluation both the speed vs. angle (ω calculated by t_2) and the time vs. angle (using t_1) values may be used to determine $I_{z,z}$. The time t_1 values are more precise in case the movement is still accelerated while the mask passes the light barrier.

Fig. 3 shows t_1 in dependance on torque T for fixed $I_{z,z}$, i.e. the moment of inertia of the turntable, and fixed $\varphi(t_1) = 215^\circ = 3.75\ \text{rad}$. The slope of the curve in the bilogarithmic plot is −0.519, compared to theoretical −0.5, since it follows from (8) that

$$t_1 = \sqrt{\frac{2\varphi(t_1) I_{z,z}}{T}}$$

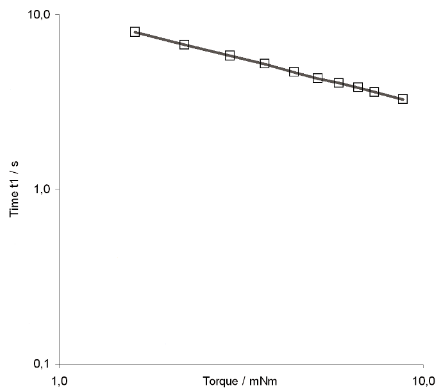

Fig 3: Bilogarithmic plot of time t_1 vs. torque T

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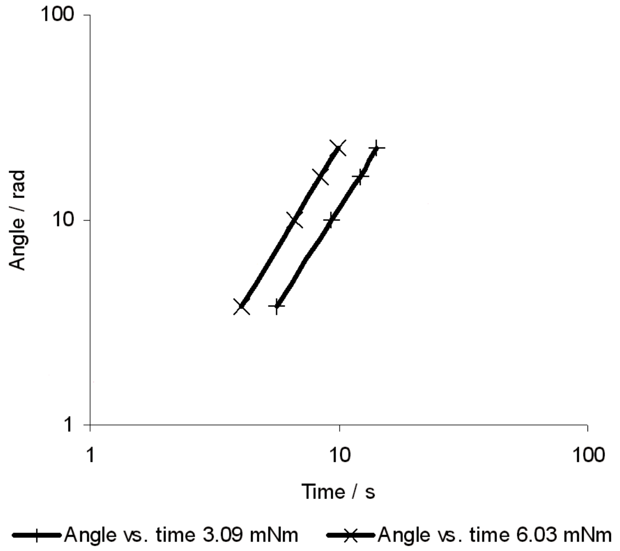
Evaluation (2/5)

Fig. 4 shows a bilogarithmic plot of angle vs. time with constant $I_{z,z}$ (of the turntable): With both values of torque the slope of the curve is nearly 2 as predicted by (8).

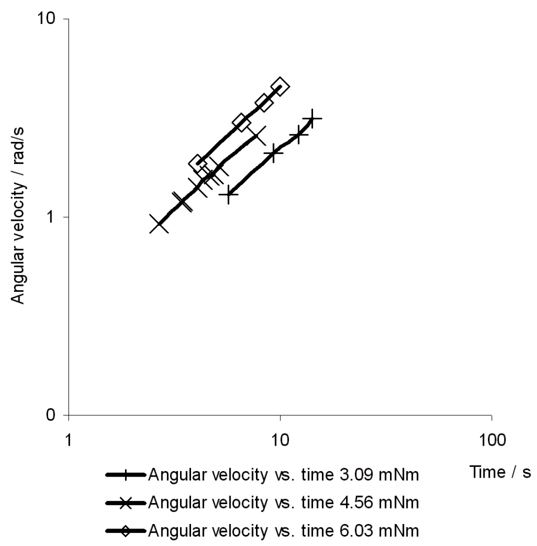
Fig 4: Bilogarithmic plot of angle φ vs. time t_1 for different torques



Evaluation (3/5)

Fig. 5 shows a bilogarithmic plot of angular velocity vs. time with constant $I_{z,z}$ (of the turntable): With all three values of torque, the slope of the curve is nearly 1 as predicted by (7).

Fig 5: Bilogarithmic plot of angle ω vs. time t_1



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Evaluation (4/5)

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Fig. 6 shows the $I_{z,z}$ values calculated from the measured t_1 values that the bar needed to reach 185°=3.23 rad with a torque applied of 3.09 mNm (r_a = 15 mm, m_a = 21 g, g = 9.81 m/s²), according to (8):

$$I_{z,z} = \frac{T \cdot t_1^2}{2\varphi}$$

Compared to the theoretical values, according to (3) $I_{z,z} = I_{\text{rod}} + m_i \cdot r_i^2$, $I_{\text{rod}} = 72$ kg cm². Two weights of 100 g each were mounted at r_i from the axis of rotation – the plot shows a good linear dependence on r_i^2 .

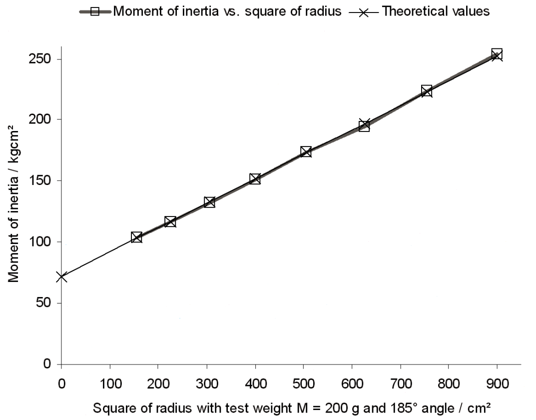

Fig 6: Measured and theoretical values of $I_{z,z}$ in dependence on square of radius

Evaluation (5/5)

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Fig. 7 shows measured $I_{z,z}$ values in dependence on the weight m_i mounted to the bar at r_i = 20 cm in comparison to theoretical $I_{z,z} = I_{\text{rod}} + m_i \cdot r_i^2$. The linear dependence on m_i can be seen.

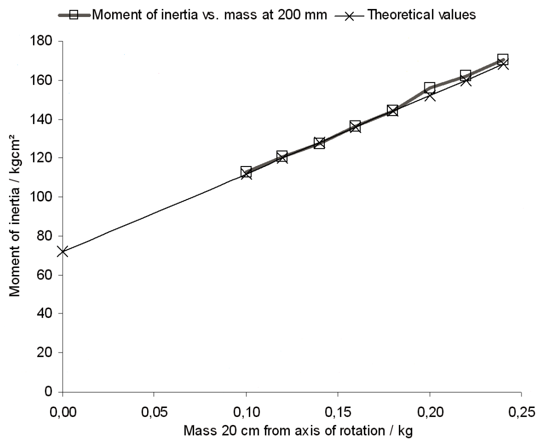

Fig 7: Measured and theoretical values of $I_{z,z}$ in dependence on mass

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