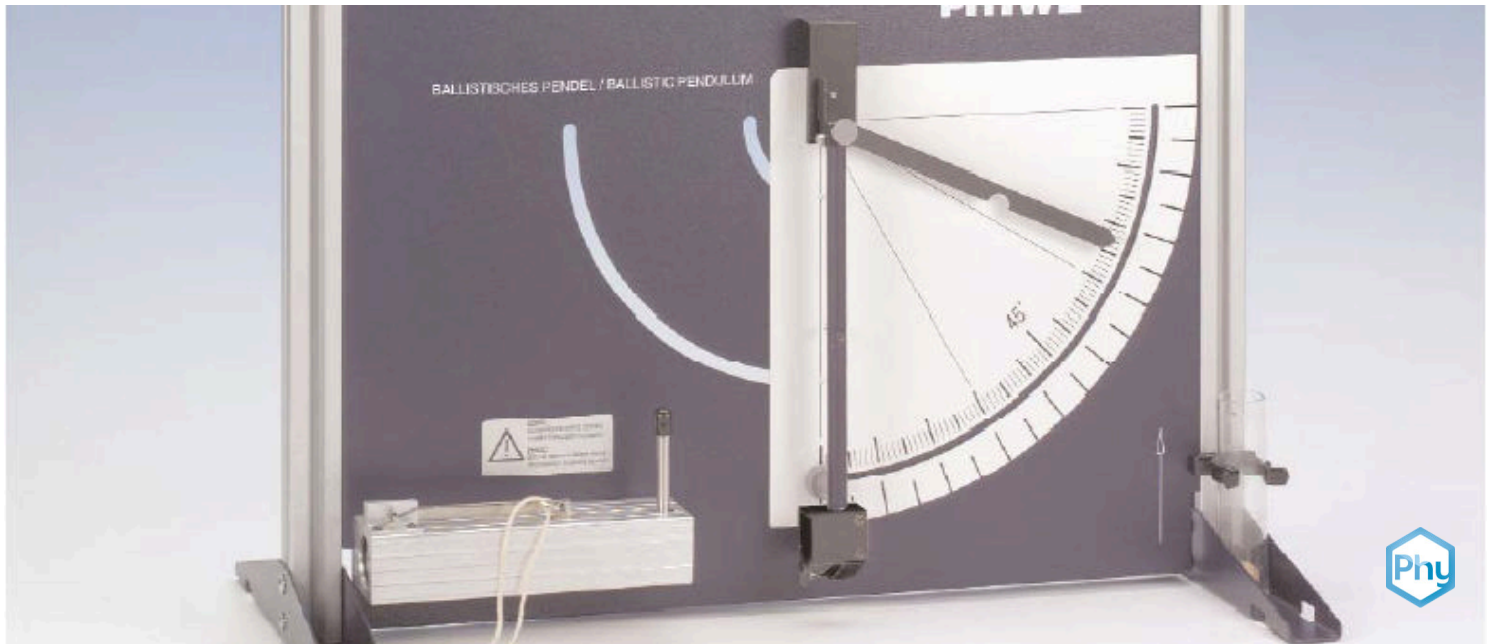


# Ballistic pendulum



The goal of this experiment is to understand the general laws of pendulum motion.

Physics

Mechanics

Dynamics & Motion



Difficulty level

hard



Group size

2



Preparation time

45+ minutes



Execution time

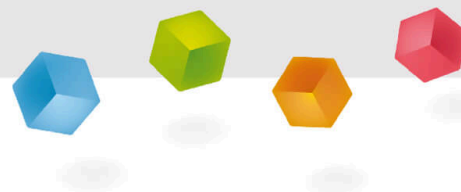
45+ minutes

This content can also be found online at:



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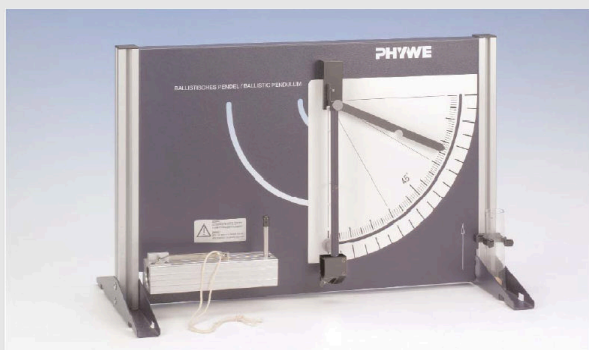
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# General information

## Application

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Experimental setup

A classic method of determining the velocity of a projectile is to shoot the projectile into a resting mass which is large compared to the projectile's mass and hung as a pendulum. In the process, the projectile remains in the pendulum mass and oscillates with it. This is an inelastic collision in which the momentum remains unchanged. If the pendulum's mechanical data are known, one can infer the velocity of the pendulum's mass (including the projectile's mass) at the lowest point of the pendulum's oscillation from the amplitude of the pendulum's oscillation. The momentum of the two masses in this phase of the oscillation must thus be equal to the impulse of the projectile before it struck the pendulum. If one knows the masses of the pendulum and the projectile, one can calculate the projectile's velocity.

## Other information (1/2)

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## Prior knowledge



There is no prior knowledge necessary.

## Main principle



In order to be able to use this measuring principle without danger, the following set-up is used here: A steel ball is shot at the mass of a pendulum with the aid of a spring catapult. The pendulum mass has a hollow space in which the steel ball is held.

If, additionally, two light barriers and a time measuring device are available, an independent, direct measurement of the initial velocity of the ball can be made.

## Other information (2/2)

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## Learning objective



The goal of this experiment is to understand the general laws of pendulum motion.

1. Measurement of the oscillation amplitudes of the ballistic pendulum after capturing the steel ball for the three possible tension energies of the throwing device.

2. Calculation of the initial velocities of the ball from the measured oscillation amplitudes.

3. Plotting of the velocity  $v$  of the steel ball as a function of the maximum deflection ( $0 \dots 90^\circ$ ) of the pendulum.

4. Determination of the correction factor  $f_{cor}$ .

5. Measure the initial velocities corresponding to the three tension steps of the throwing device by performing 10 measurements each with subsequent mean value calculation.

## Tasks



## Theory (1/3)

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For the evaluation, we proceed from the approximation that the pendulum's mass  $M$  and the Mass  $m$  of the captured ball move as one mass point at the location of their common centre of gravity. If we then set the potential energy of the pendulum in its resting position to zero, the following is valid for the potential energy at the highest point of the oscillation:

$$E_{\text{pot}} = (m + M) \cdot g \cdot \Delta h$$

where  $g$  is the acceleration of gravity and  $\Delta h$  the height by which the centre of gravity was raised. In Fig. 1 one sees that with  $r$  as the distance between the pivot point and the centre of gravity, one can also write this formula as:

$$E_{\text{pot}} = (m + M) \cdot g \cdot (1 - \cos \varphi) \quad (1)$$

## Theory (2/3)

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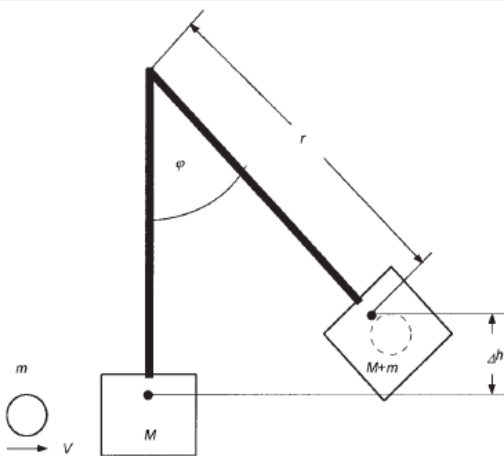


Fig. 1: Diagram on the theory of the ballistic pendulum.

This potential energy must be equal to the kinetic energy  $E_{\text{kin}}$  immediately after the collision:

$$E_{\text{kin}} = \frac{1}{2} \cdot (m + M) \cdot v_p^2 \quad (2)$$

where  $v_p$  is the velocity of the pendulum's mass (including the ball) immediately after the collision. If one substitutes the momentum  $p = (m + M) \cdot v_p$ , in this equation, one obtains the following:

$$E_{\text{kin}} = \frac{p^2}{2 \cdot (m + M)}$$

$$\text{or } p = \sqrt{2 \cdot (m + M) \cdot E_{\text{kin}}}$$

## Theory (3/3)

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Before the collision, the pendulum was at rest. Due to the principle of conservation of momentum, the momentum  $p = m \cdot v$  must be equal in magnitude to the momentum of the ball before the collision. One obtains the following for the velocity  $v$  of the ball before the start, i.e. for the parameter to be determined:

$$v = \frac{m+M}{m} \cdot \sqrt{2 \cdot g \cdot r \cdot (1 - \cos \varphi)} \quad (3)$$

## Equipment

Position	Material	Item No.	Quantity
1	Ballistic Unit	11229-10	1
2	Ballistic Pendulum, f. Ballist. Unit	11229-20	1
3	Steel ball, d = 19 mm	02502-01	2
4	Speed measuring attachment	11229-30	1
5	Power supply 5 VDC/4 A, according EN61558-2-16	12651-99	1

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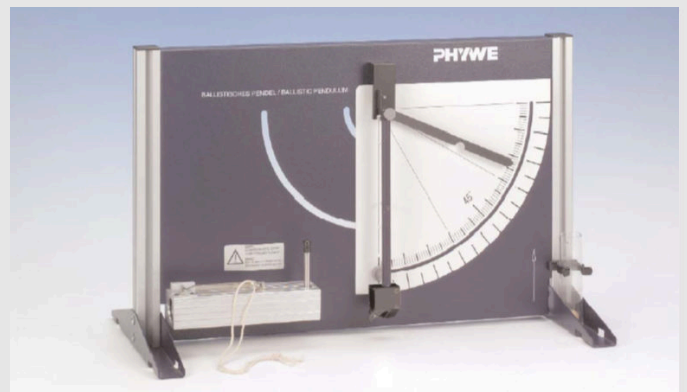
# Setup and Procedure

## Setup and procedure (1/2)

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Assemble the device according to Fig. 1. It must be standing on a stable table during the measurements.

Information on the operation of the ballistic pendulum is to be obtained from the appropriate operating instructions.



Assemble the device according to this figure from the page "Application".

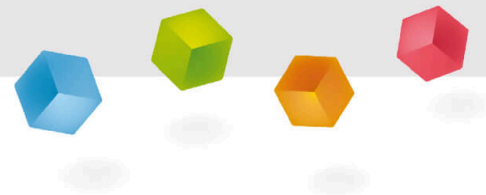
## Setup and procedure (2/2)

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Before stretching the spring of the throwing device, affix the steel ball to the holding magnet of the bolt. Then pull the bolt back until the desired lock-in position has been reached. Now, without touching the pendulum's pulling pin, ensure that the pendulum is at rest and that the trailing pointer indicates nearly zero. After these preparations have been completed, trigger the shot by pulling the release lever. The amplitude of the pendulum's oscillation can be read from the trailing pointer.

Note: Due to the friction involved in the functioning of the trailing pointer, the measured amplitude can be slightly low. It is therefore advisable to shoot at the resting pendulum using the same spring tension for a second and a third time without resetting the trailing pointer. When the trailing pointer is not moved any further, one can assume that the angle indicated has not been falsified by friction.

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## Evaluation



## Evaluation (1/6)

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Fig. 2 shows the function  $v(\varphi)$ . Between 0 and 90°, the curve deviates only slightly from a straight line, i. e. the deflection of the pendulum is, as a good approximation, proportional to the velocity of the ball.

For the evaluation, determine the mass  $m$  of the ball. The position of the centre of gravity with captured steel ball is marked on the pendulum's body. The mass  $M$  of the pendulum's body can only be determined together with its mounting. The exact value is slightly less and can be obtained from the operating instructions.

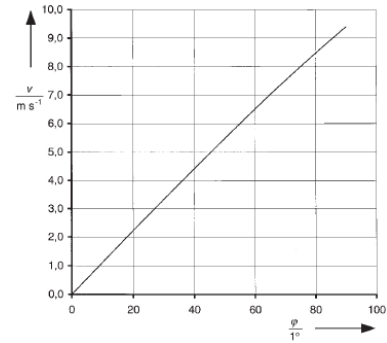


Fig. 2: Theoretical calibration curve for a velocity measurement with a ballistic pendulum. This is only an exemplary curve as the manufacturer reserves the right to make technical alterations with regard to the properties of the device.

## Evaluation (2/6)

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### Information for an exact evaluation of the experiment

The theoretical treatment of the processes occurring on the ballistic pendulum, given above, is based on a certain idealisation. Formula (3) is consequently only approximately valid, but is very useful for practical purposes. At this time, we only wish to briefly show how one can achieve an exact evaluation. To begin with, Equation (2) for the kinetic energy is to be replaced by the equation for the rotational energy of a physical pendulum:

$$E_{\text{kin}} = \frac{1}{2} \cdot I \cdot \omega^2 \quad (2a)$$

where  $I$  is the moment of inertia of the pendulum with captured ball and  $\omega$  the angular velocity. If one substitutes the angular momentum  $L = I \cdot \omega$ , the following is obtained.

## Evaluation (3/6)

PHYWE

$$E_{\text{kin}} = \frac{L^2}{2I} \quad \text{or} \quad L = \sqrt{2I \cdot E_{\text{kin}}}$$

This angular momentum must be equal to the angular momentum  $L_b$  of the ball before the collision with reference to the pivot point of the pendulum. If  $r_b$  is the distance of the ball from the pivot point at the instant of capture, then

$$L_b = m \cdot r_b^2 \cdot \omega_b = m \cdot r_b \cdot v$$

By setting the two angular momentums equal to each other, one obtains

$$v = \frac{1}{m \cdot r_b} \cdot \sqrt{2 \cdot I \cdot (m + M) \cdot g \cdot r \cdot (1 - \cos \varphi)}$$

## Evaluation (4/6)

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The determination of the moment of inertia  $I$  of a physical pendulum is achieved by measuring its oscillation period  $T$ . For our pendulum the following results:

$$I = \frac{(m+M) \cdot g \cdot r \cdot T^2}{4\pi^2}$$

$$\text{and thus } \frac{m+M}{m} \cdot \left(\frac{r}{r_b}\right) \cdot \left(\frac{g \cdot T}{2\pi}\right) \cdot \sqrt{2 \cdot (1 - \cos \varphi)} \quad (3a)$$

If we now term the velocity calculated with the approximation formula (3)  $v_{\text{app}}$  and the exact value calculated with the exact formula (3a)  $v_{\text{ex}}$ . The two differ in accordance with  $v_{\text{ex}} = f_{\text{cor}} \cdot v_{\text{app}}$  by the correction factor:

$$f_{\text{cor}} = \frac{T}{2\pi \cdot r_b} \cdot \sqrt{r \cdot g}$$

## Evaluation (5/6)

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This formula becomes more illustrative if one substitutes the following expression for  $T$ , in which  $r_m$  is the length of a mathematical pendulum having the oscillation period  $T$ :

$$T = 2\pi \cdot \sqrt{\frac{r_m}{g}}$$

The following is now obtained:

$$f_{\text{cor}} = \frac{\sqrt{r \cdot r_m}}{r_b} \quad (4)$$

## Evaluation (6/6)

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For an imaginary experiment with a projectile in the form of a mass point, which collides inelastically with a mathematical pendulum at rest, the three lengths used in Formula (4) are equal, i.e.  $f_{\text{cor}} = 1$ .

By measuring  $r$  and  $r_b$  for the physical pendulum used as well as by calculating  $r_m$  from the measured oscillation period, one can easily determine the correction factor  $f_{\text{cor}}$  for the velocities obtained with Equation (3) from Equation (4).



Fig. 3: Experimental set-up with supplement for direct measurement of the initial velocity of the ball.